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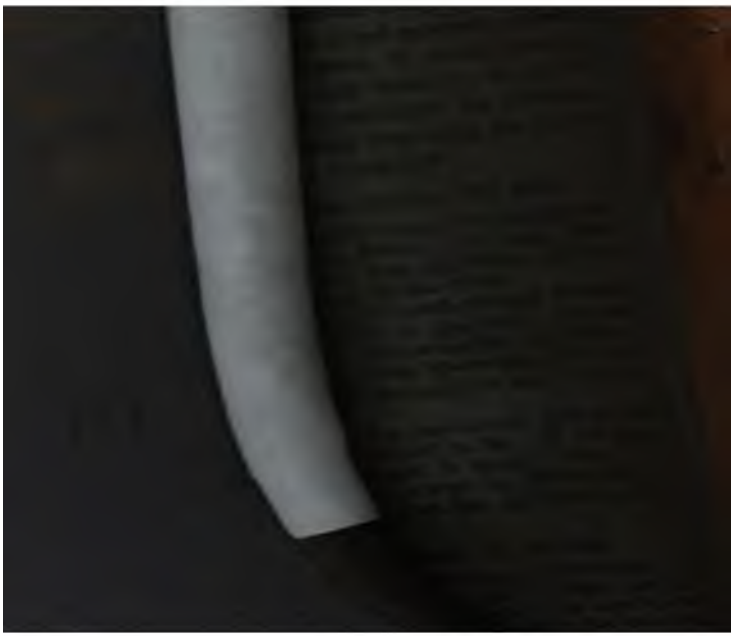

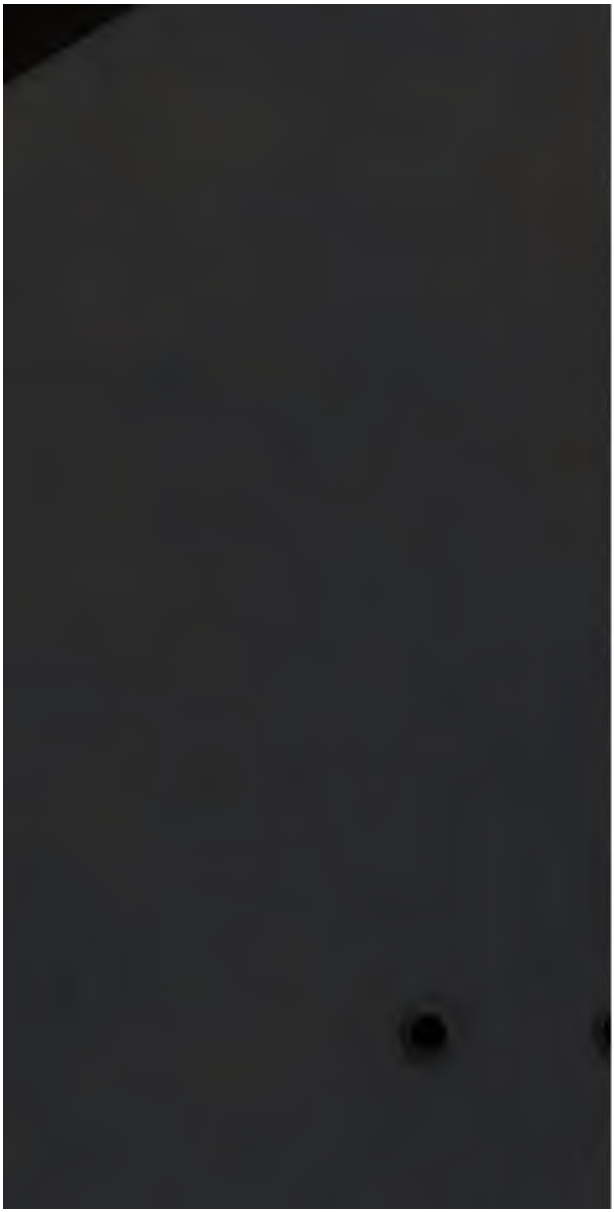

WELCH'S IMPROVED

AMERICAN ARITHMETIC,

ADAPTED TO THE

CURRENCY OF THE UNITED STATES

TO WHICH IS ADDED

A CONCISE TREATISE

ON THE

MENSURATION OF PLANES AND SOLIDS.

COMPILED BY

OLIVER WELCH.

IMPROVED AND ILLUSTRATED WITH QUESTIONS ADAPTED TO THE RULES, FOR THE USE OF

SCHOOLS AND ACADEMIES.

BY AN EXPERIENCED TEACHER.

BOSTON:
PUBLISHED BY LINCOLN, EDMANDS AND CO.
1834.

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23 August 1858

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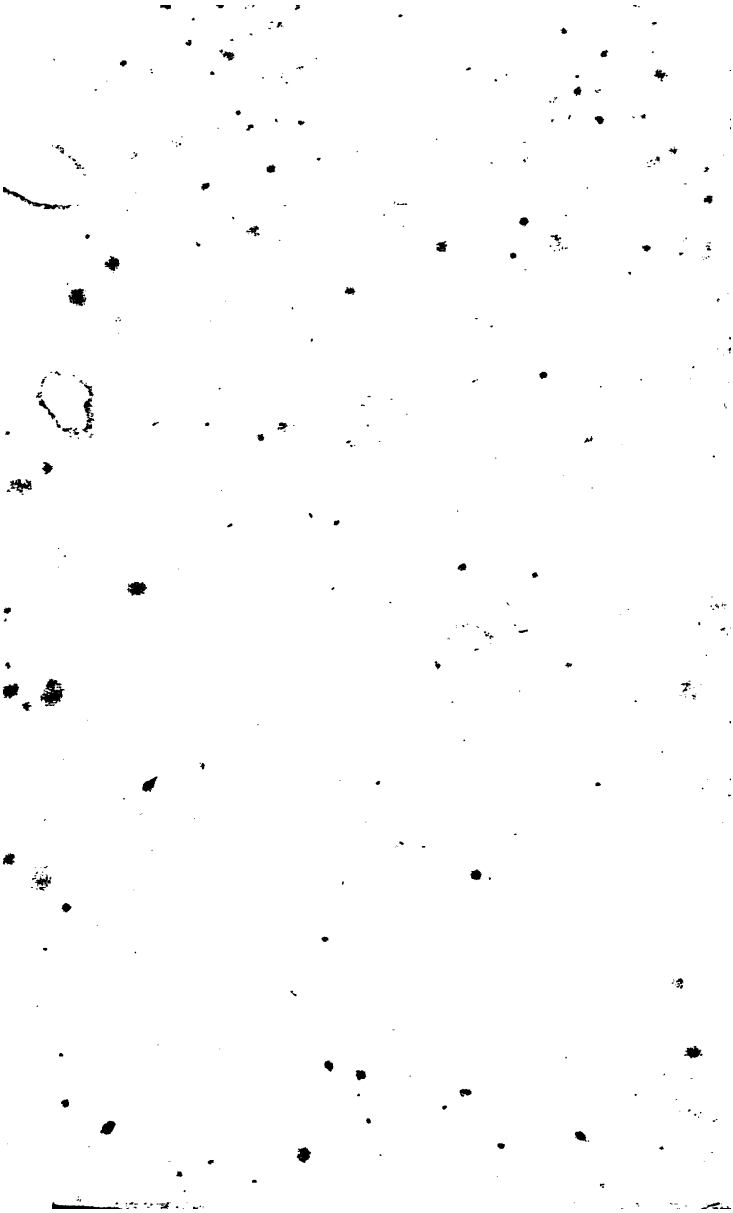

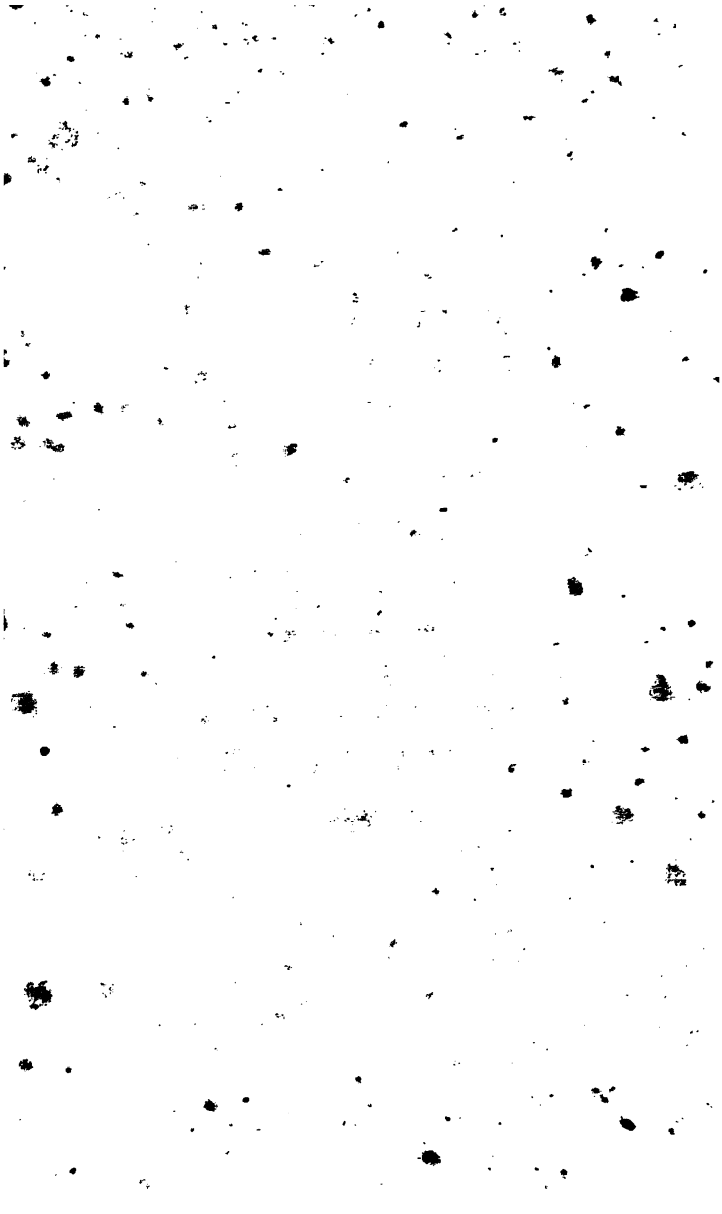

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To meet the wishes of many instructers, a short COMPEND OF MENTAL ARITHMETIC has been prefixed to the book, in the hope that it will now be found to contain all that is necessary. The compend is confined to simple elementary rules, in order to furnish an introduction to the study.

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PREFACE

TO THE FIRST EDITION OF

WELCH'S AMERICAN ARITHMETIC REVISED.

Considering the encouragement with which the original work has met, since its first appearance to the public, the author feels assured that a like system was wanted, and will still receive the entire approbation of a generous public.

"If a quick sale and continued demand of a book are recommendations, this system of Arithmetic may claim a competent share of them; for since the first publication of the original, not less than 26000 copies have been published and sold, and yet an increasing demand." No other recommendations than these and its own merits will be offered in this edition: desirous to increase the benefit of the work, the author has made many important additions and corrections and omitted all matter which was deemed superfluous in the original. In revising the work, it was thought proper and beneficial that additions to the rules of Interest and to the method of casting interest on Notes, Bonds, &c. should be made, and also that it should comprise a few valuable rules which have never appeared in the original, viz.—*Compound Average Judgment, Commission, Brokerage, Insurance, Barter, Loss and Gain, Single and Double Position, Arithmetical and Geometrical Progression;* and a series of useful *Miscellaneous Questions;* all arranged and well calculated to promote instruction in this branch of science.

The Revisor having been for many years engaged in instructing, found that a thorough knowledge of the *Rule* previous to the operation of the *Example*, was highly requisite before any progress could be made; and that the *easiest method* to simplify and make it familiar was far the *best*. *This is presumed to be accomplished.*

Annexed to each *Rule* are *interrogatories* numbered by figures in their order, and these figures referring to a like, in the Rule, where the answer to the *interrogative* should commence; and when these are once perfectly committed to memory, all obscurity is removed and the Rule then becomes easy and plain and needs but little or no further explanation.

As this work is now presented to the public, the author has full confidence in commending it to instructers, well assured it will lessen their own labor and facilitate the improvement of their pupils. It is perfectly obvious, that great convenience will be found both by instructers and scholars in having the interrogatives printed on the same page with the *rule* from which they are to be answered, as no labor is required in selecting appropriate answers by reading and studying the whole rule. This system, it is hoped, will supply the long sought for *Classical Arithmetic*, whereby whole schools can be examined, collectively or separately; and while the author expresses his confidence of the merits of such a work, he trusts that the following pages will sufficiently recommend themselves.

Northwood, N. H. 1833.

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FIRST LESSONS

IN

ARITHMETIC.

LESSON FIRST.*

A thing standing alone, as a ball, counts	● One.
If another thing be put with it, they both count ●	● Two.
If two things be put with it, they all count ●●	● Three.
If three things be put with it, they count ●●●	● Four.
If four things, they then count ●●●●	● Five.
If five things, they then count ●●●●●	● Six.
If six things, they then count ●●●●●●	● Seven.
If seven things, they count ●●●●●●●	● Eight.
If eight, they then count ●●●●●●●●	● Nine.
If nine, they count ●●●●●●●●●	● Ten.

The pupils must be taught to count the balls in different directions, and to count up the total of the four upper lines of balls, the object of the lesson being to learn them to count as far as ten.

LESSON SECOND.*

		Ten,		
One..... ●	and	●●●●●●●●●●	count	Eleven.
Two..... ●●	and	●●●●●●●●●●	count	Twelve.
Three..... ●●●	and	●●●●●●●●●●	count	Thirteen.
Four..... ●●●●	and	●●●●●●●●●●	count	Fourteen.
Five..... ●●●●●	and	●●●●●●●●●●	count	Fifteen.
Six..... ●●●●●●	and	●●●●●●●●●●	count	Sixteen.
Seven.... ●●●●●●●	and	●●●●●●●●●●	count	Seventeen.
Eight.. ●●●●●●●●	and	●●●●●●●●●●	count	Eighteen.
Nine..●●●●●●●●●	and	●●●●●●●●●●	count	Nineteen.
Ten ●●●●●●●●●●	and	●●●●●●●●●●	count	Twenty.

* The lessons thus marked * should be taught to classes by ocular demonstration, by means of a large slate, or black board, placed in some conspicuous part of the school-room. When an individual is to be instructed, the book itself can be used, while the pupil is learning and reciting. Pupils must understand thoroughly before passing forward. The lessons can afterwards be recited without the book or slate.

LESSON THIRD.*

TEN ●●●●●●●●●● and
TEN ●●●●●●●●●● make } TWENTY, { if both lines be counted.

If there be counted, with the twenty above,

ONE ● they all make.....TWENTY-ONE.
TWO ●● they all make.....TWENTY-TWO.
THREE ●●● they all make.....TWENTY-THREE.
FOUR ●●●● they all make.....TWENTY-FOUR.
FIVE ●●●●● they all make.....TWENTY-FIVE.
SIX ●●●●●● they all make.....TWENTY-SIX.
SEVEN ●●●●●●● they all make.....TWENTY-SEVEN.
EIGHT ●●●●●●●● they all make....TWENTY-EIGHT.
NINE ●●●●●●●●● they all make .TWENTY-NINE.
TEN ●●●●●●●●●● they all make THIRTY.†

THIRTY-ONE.
THIRTY-TWO.
THIRTY-THREE.
THIRTY-FOUR.
THIRTY-FIVE.
THIRTY-SIX.
THIRTY-SEVEN.
THIRTY-EIGHT.
THIRTY-NINE.
FORTY.
FORTY-ONE.
FORTY-TWO.
FORTY-THREE.
FORTY-FOUR.
FORTY-FIVE.
FORTY-SIX.
FORTY-SEVEN.
FORTY-EIGHT.
FORTY-NINE.
FIFTY.
FIFTY-ONE.
FIFTY-TWO.
FIFTY-THREE.
FIFTY-FOUR.
FIFTY-FIVE.
FIFTY-SIX.
FIFTY-SEVEN.
FIFTY-EIGHT.
FIFTY-NINE.
SIXTY.
SIXTY-ONE.
SIXTY-TWO.
SIXTY-THREE.
SIXTY-FOUR.
SIXTY-FIVE.
SIXTY-SIX.
SIXTY-SEVEN.
SIXTY-EIGHT.
SIXTY-NINE.
SEVENTY.
SEVENTY-ONE.
SEVENTY-TWO.
SEVENTY-THREE.
SEVENTY-FOUR.
SEVENTY-FIVE.
SEVENTY-SIX.
SEVENTY-SEVEN.
SEVENTY-EIGHT.
SEVENTY-NINE.
EIGHTY.
EIGHTY-ONE.
EIGHTY-TWO.
EIGHTY-THREE.
EIGHTY-FOUR.
EIGHTY-FIVE.
EIGHTY-SIX.
EIGHTY-SEVEN.
EIGHTY-EIGHT.
EIGHTY-NINE.
NINETY.
NINETY-ONE.
NINETY-TWO.
NINETY-THREE.
NINETY-FOUR.
NINETY-FIVE.
NINETY-SIX.
NINETY-SEVEN.
NINETY-EIGHT.
NINETY-NINE.
ONE HUNDRED.

† The instructer should follow up this plan of illustration, (from twenty-one to thirty,) as far as one hundred, or farther, if he thinks proper.

ADDITION.

LESSON FOURTH.*

How many balls are two balls and three balls?
How many balls are three balls and two balls?
Two balls and three balls are how many balls?
Three balls and two balls are how many balls?
How many balls are two balls and four balls?
Four balls and two balls are how many balls?
Three balls and three balls are how many balls?
How many balls are two balls and five balls?
How many balls are five balls and three balls?
How many balls are four balls and three balls?
How many balls are six balls and two balls?
Seven balls and three balls are how many balls?
Eight balls and two balls are how many balls?
How many balls are six balls and four balls?
How many balls are five balls and five balls?
Henry has three balls, and George has five balls; how many balls have both?
James lost four balls, and John lost five balls; how many balls did both lose?

LESSON FIFTH.

To be recited without the book or slate.

One and two are how many?
Two and one are how many?
Two and three are how many?
Three and two are how many?
Two and four are how many?
Three and three are how many?
Two and five are how many?
Five and three are how many?
Four and three are how many?
Three and five are how many?
Four and five are how many?
Six and two are how many?
Seven and three are how many?
Eight and three are how many?

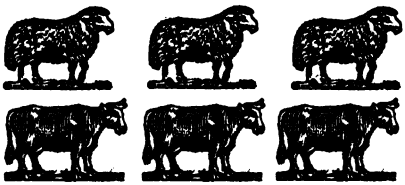
After a full explanation, the scholars should be made to recite from the lessons of this nature, without the illustrations upon the black board.

LESSON SIXTH.

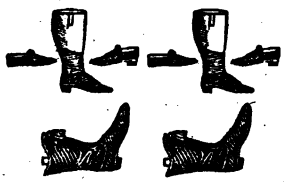
A man led his horse to drink, and two horses followed; how many horses were there in the whole?

If there be two hats in one place, and three trees in another place, how many hats and trees are there in both places?

A farmer's son sheared three sheep, and drove three cows to pasture; how many cows and sheep were there in all?



One shoemaker made two boots and four shoes in a week, and another made two boots in a week; how many boots and shoes did both make?



LESSON SEVENTH.

Charles has three cents, and George has five cents; how many have both?

Three and five are how many?

John gave three cents for an orange, and seven cents for a top; how many cents did he give for both?

Three and seven are how many?

Charles's father gave him three cents at one time, and nine at another; how many did he have?

How many are three and nine?

If there are four scholars in the first class, and seven in the second, how many are in both classes?

How many are four and seven?

John found, while walking to school, six walnuts, and while coming home he found five more; how many did he find in all?

Six and five are how many?

If the schoolmaster punished four scholars in one week, and six in another, how many did he punish in both weeks?

Four and six are how many?

George's father owned four horses, and Charles's father owned eight; they put them all into one stable; how many were there in the stable?

Add four to eight, and how many will it make?

Levi used seven pens in one week, and nine in another week; how many did he use in both weeks?

Seven and eight are how many?

Add seven and nine together, and what will the amount be?

A boy skated two miles in one day, three miles on another day, and four miles on the third day; how many miles did he skate in all?

Two, three and four are how many?

Three boys, five girls and two men were in a sleigh together; how many persons were there in all?

Three, five and two are how many?

Isaac went out to fish, and caught four fishes; Jacob went into the woods, and shot six rabbits; and their father went to the poultry yard, and killed three chickens: all these were put on the table for dinner: how many were there in all?

Three, six and four are how many?

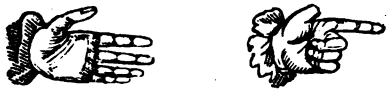
If there be in a barn six horses, five cows, and seven sheep, how many animals are there in all?

Six, five and seven are how many?

A stage drawn by six horses, a coach by two, and a chaise by one, passed by; how many horses were there in all?

SUBTRACTION.

LESSON EIGHTH.*



The fingers and thumb on one hand count how many?

If you take away three fingers and the thumb by shutting them up, how many remain open?

Now, if you shut up that one also, how many will remain open?

Take one from one, and how many remain?

Take four from five, and how many remain?

Take two balls from three balls, and how many remain?

From three balls take two balls, and how many remain?

From five balls take two balls, and how many remain?

Take three balls from five balls, and how many remain?

Take three balls from seven balls, and how many remain?

From eight balls take three balls, and how many remain?

Take four balls from seven balls, and how many remain?

Take five balls from ten balls, and how many remain?

From ten balls take four balls, and how many remain?

From nine balls take six balls, and how many remain?

Take from ten balls, three balls, and how many remain?

Take seven balls from nine balls, and how many remain?

Charles had in his pocket ten marbles; he took out six and gave them to his brother; how many had he left?

The following may be recited without the black board.

Two from three leave how many?

Two from four leave how many?

Two from five leave how many?

Three from five leave how many?

Three from four leave how many?

Four from five leave how many?

Two from six leave how many?

Five from eight leave how many?

Six from nine leave how many?

Seven from ten leave how many?

Five from ten leave how many?

Six from ten leave how many?

LESSON NINTH.

If you buy a primer for four cents, and give the bookseller ten cents, how many cents should he give you in change?

If Charles buy an orange for five cents, and sell it for six cents, how many cents does he gain?

George had ten walnuts, and gave three of them to Caleb; how many walnuts had he left?

If you have twelve cents, and pay seven of them for candy, how many cents will you have left?

A man shot at fifteen birds, and killed all but six; how many birds did he kill?

Fourteen boys were playing at football; six of them went home, tired; how many were left at play?

Seven from twelve leave how many?

At the end of a quarter, eleven boys left school; and at the beginning of the next quarter, seventeen new pupils joined the school: how many scholars did the master gain?

Jacob has twenty buttons on his coat, and seven on his vest; how many more had he on his coat than he had on his vest?

A farmer gathered from his orchard last year twenty-one barrels of apples; the present year he had but fifteen; what was the amount of difference?

If you should buy twenty-five pounds of raisins, and the grocer should send them home in two bundles, one of which should weigh thirteen pounds, how many pounds ought the other parcel to weigh?

If nineteen persons be in a room, and only four of them be sitting, how many of them are standing?

Seventeen men, sailing in a boat, were overset, and five of them were drowned; how many were saved?

Eighteen crows were caught in a net, and four of them escaped while the farmer was trying to take them from it; how many did the farmer get?

A man owed twenty-six dollars, and paid seven; how many dollars did he then owe?

Sarah made sixteen handkerchiefs, and gave four of them to her friends; how many had she left?

Caroline carried a bunch of twenty-five quills to learn to make pens, and she cut up nine in learning; how many had she left?

Charles learned ten chapters in the Bible, and Lydia learned eighteen; how many more did Lydia learn than Charles?

LESSON TENTH.

In Addition and Subtraction combined.

A canal boat took from a pile of twelve bales of cotton four bales into the fore part and three bales in the aft part; how many bales were left in the pile?

One man owed another twenty-three dollars; he paid him at one time seven dollars, and at another time six dollars; how much remained due after both payments?

A stage went twenty-one miles in three hours; the second hour it went seven miles, and the third hour it went six miles; how many miles did it go the first hour?

Take eight and nine from twenty-five, and what remains?

Susan had made nineteen pincushions for sale at fairs; she gave five to a fair for the Sabbath School, and nine to the fair for the benefit of the poor; how many had she left to give away?

A boy had seventeen cents to spend on his birthday, and bought six cents' worth of gingerbread and three cents' worth of apples; the rest of the money he spent for candy; how many cents' worth of candy did he buy?

A pupil agreed with his master to learn fifteen lessons in Geography in three weeks; the first week he learned seven, and the second week six; how many had he to learn the third week?

A girl had a week given her to learn six lessons in Arithmetic and four lessons in Astronomy; she recited well in five of the lessons in Arithmetic and two of the lessons in Astronomy; how many lessons was she deficient in?

Charles had five marbles, and James had eight; but in going home from school, one lost three, and the other lost four: how many did they both have left?

Nine and four from ten and five leave how many?

Ten and five from eleven and seven leave how many?

To seven add two and four, then subtract five, and how many remain?

MULTIPLICATION.

LESSON ELEVENTH.*

Joseph's master told him he would give him a top every week that he would recite his lessons correctly during the month; Joseph recited all his lessons well only one week; how many tops did he receive?

One times one is what number?

How many balls are one time two balls?

How many balls are two times one ball?

How many balls are two times two balls?

How many balls are two times three balls?

How many balls are two times four balls?

How many balls are two times five balls?

How many balls are two times six balls?

How many balls are two times seven balls?

How many balls are two times eight balls?

How many balls are two times nine balls?

How many balls are two times ten balls?

How many are three times two? three times three? three times four? three times five? three times six? three times seven? three times eight? three times nine? three times ten?

How many are four times two? four times three? four times four? four times five? four times six? four times seven? four times eight? four times nine? four times ten?

How many are five times two? five times three? five times four? five times five? five times six? five times seven? five times eight? five times nine? five times ten?

How many are six times two? six times three? six times four? six times five? six times six? six times seven? six times eight? six times nine? six times ten?

How many are seven times two? seven times three? seven times four? seven times five? seven times six? seven times seven? seven times eight? seven times nine? seven times ten?

How many are eight times two? eight times three? eight times four? eight times five? eight times six? eight times seven? eight times eight? eight times nine? eight times ten?

How many are nine times two? nine times three? nine times four? nine times five? nine times six? nine times seven? nine times eight? nine times nine? nine times ten?

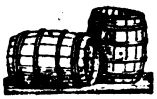
How many are ten times two? ten times three? ten times four? ten times five? ten times six? ten times seven? ten times eight? ten times nine? ten times ten?

LESSON TWELFTH.

If one ship has three masts, how many masts are there in three ships?

Three times three are how many?

If one barrel has six hoops, how many hoops are there upon two barrels?

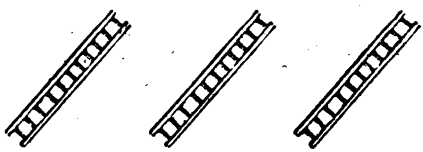


How many are twice six?

If two houses have ten windows each, how many will both have?

Two times ten are how many?

If a ladder has eleven rounds, how many rounds has three ladders of the same length?



Three times eleven are how many?

If eleven be multiplied by three, what will be the amount?

What number will be produced by multiplying twelve by three?

If one apple cost two cents, what will twelve apples cost? What will five cost? What will seven cost?

If a horse travel seven miles in one hour, how far will he go if he travel four hours?

If there are eight forms in the school, and each form have nine seats, how many scholars can be seated?

If the school keeps six hours every day, how many hours will a pupil be in school during six days?

A quarter of a dollar is twenty-five cents; how many cents are two quarters? three quarters? four quarters?

An eagle is ten dollars; how many dollars are there in ten eagles? in fifteen eagles? in eighteen eagles?

If there are seven days in one week, how many days are there in four weeks?

DIVISION.

LESSON THIRTEENTH.*

Divide two balls between two boys, and what number will each boy have?

How many times is one contained in two?

Divide four balls between two boys, and how many balls will each boy have?

Two is contained how many times in four?

Divide six balls between three boys, and how many balls will each boy receive?

Three is contained in six how many times?

Divide six balls between two boys, and how many will each have?

Two is contained how many times in six?

Divide eight marbles between two boys, and how many will each get?

What number will be obtained by dividing eight by two?

If nine marbles be equally divided between three boys, how many will each boy have?

Three is in nine how many times?

Suppose ten balls were to be given equally to five boys, how many ought each one to have?

Five is in ten how many times?

Two pears cost two cents; how much did one cost?

Two oranges cost four cents; how much did each one cost?

Three apples cost six cents; how much did each cost?

Two lemons cost six cents; how much did one cost?

Two tops cost eight cents; how much did one cost?

Three pencils cost nine cents; how much did each cost?

Five quills cost ten cents; how much did they cost apiece?

In four how many times two?

In six how many times three? how many times two?

In eight how many times two? how many times four?

In nine how many times one? how many times three?

In ten how many times two? how many times five?

How many times is three contained in fifteen? in eighteen? in twenty-one? in thirty?

Ten mills make one cent; how many cents, then, are there in one hundred mills? in fifty mills?

LESSON FOURTEENTH.

If twenty-five cents be put in five equal piles, how many cents will there be in each pile?

Five is contained in twenty-five how many times?

If Charles owes John thirty cents, and pays him two cents each day, how long will it take him to pay the whole?

Thirty divided by two gives what number?

Three girls agree to give a present to their instructer which will cost them thirty-three cents; what will each girl have to pay?

Three is in thirty-three how many times?

A Sunday school, containing forty scholars, go every Sabbath in procession from the vestry to the church; how many sections of them will there be if they walk four together? how many if they walk two together?

How many times is four contained in forty?

If Sarah has forty-five apples, and eats nine every week as long as they last, how many weeks will she be in eating them all?

How many times is nine contained in forty-five?

Caroline's mother made thirty-six pies for Thanksgiving, and two of them were ate at dinner each day while they lasted; how long did it take to eat all?

Two is contained how many times in thirty-six?

Divide thirty-six nuts among twelve boys, and how many will each boy have?

Twelve is contained in thirty-six how many times?

A class had a book of forty-eight pages to read through; the master gave them six pages to read each day; how many days did it take them to get through the book?

How many times is six contained in forty-eight?

A farmer sent sixty-six bushels of corn to market, and his agent happened to sell just eleven bushels each day; how long did it take him to sell the whole?

Sixty-six contains how many times eleven?

Divide fifty cents among ten boys, and how many cents will each have?

Divide seventy-five cents among three boys, and how many cents will each have?

A book contains sixty-six lessons, and a class of boys recited eleven of them each week; how long did it take them to get through the book?

LESSON FIFTEENTH.

Multiplication and Division combined.

A father gave his two sons five cents on Saturday and on Monday, to be divided equally between them; how many cents would each have?

James went out two afternoons to pick berries, and obtained three pints each time, and he divided them equally between his father and mother; how many pints did each have?

A butcher went to market three days, and each day bought six cattle; he killed two each day: how many days did it take him to kill the whole?

Two boys bought five oranges for four cents apiece; how much did each boy pay?

If six times six cherries be divided among nine girls, how many will each have?

If seven times nine apples be divided among three girls, how many will be the share of each?

Nine times six divided by three times nine give how many?

LESSON SIXTEENTH.

Addition, Subtraction, Multiplication and Division combined.

One boy had two nuts, another had three nuts; they put them away to dry, and a rat ate up one; how many were there left for each boy?

Charles's father gave him two cents for every week that he brought home from school a reward of merit, which he did for three weeks; and because Charles had tried hard to do the same the next week, he gave him three cents more: Charles spent one cent for candy, and divided the rest among himself and three sisters; how many cents did each have?

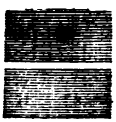
Add together seven and five, from the sum subtract three, multiply the remainder by two, and divide the product by three, and what will be the result?

There were in a school eight girls and ten boys at the close of a quarter; at the commencement of a new quarter, it was found that seven girls and four boys more were added to the school; the master sent away five the first week for bad behavior: there were four benches in the school; how many scholars sat on each bench?

FRACTIONS.

LESSON SEVENTEENTH.*

A Fraction is a part of any whole thing. When the parts into which a thing is cut or divided are just alike, in size or value, they are said to be equal.



Any thing, as, for instance, a square piece of paper, which has been cut into two equal parts, is said to be divided into halves.

It makes no difference in what direction it be cut, if the parts be equal.

How many halves are there in a whole thing?

If a piece of paper be cut into four equal parts, it is divided into fourths, or quarters.

How many quarters are there in a whole thing?
How many quarters are there in half a thing?
What part of a whole thing is a quarter?

If a thing be divided into eight equal parts, each part is called an eighth.

How many eighths are there in a quarter?
How many in a half? In three quarters?
How many in a whole thing?

A thing cut into three equal parts is said to be divided into thirds.

Thus, if James should divide a piece of candy equally between himself and two schoolmates, they would each have a third part.

If a thing be cut into six equal parts, each part is called a sixth.

How many sixths are there in a third?
How many sixths are there in two thirds?
How many sixths are there in a half?
How many sixths are there in a whole thing?

If a thing be cut into nine equal parts, each part is called a ninth.

How many ninths are there in a third?
How many in two thirds?
How many in a whole thing?

LESSON EIGHTEENTH.*

If a boy has eight marbles, and loses four, what part or proportion does he have left?

If he loses two, what proportion does he lose? What proportion does he have left?

If he loses one, what part of the whole does he lose? What part will he then have left?

What part of eight is seven? What part is six? What part is four? What part is one? What part is two? What part is five? What part is three?

Is there any difference between two eighths and one quarter? Or between four eighths and a half? Or six eighths and three quarters?

Note.—The instructer should here teach the pupils to give the lowest terms.

What part of nine is three?

What proportion of nine marbles is six?

What part of six is three? What part of six is two? What part of six is one? What part of nine is one? What part of nine is six? A third of nine is how many? Two thirds of nine are how many?

What part of ten is two? What part is six? What part is five? What part is four? What part is eight? What part is three? What part is seven?

Four fifths of ten are how many?

Three fifths of ten are how many?

Two fifths of ten are how many?

Five fifths is what proportion of ten?

If a sheet of gingerbread is worth ten cents, how much is one fifth of it worth? What is two fifths of it worth?

When wood is seven dollars a cord, what proportion of a cord can you buy for one dollar? What proportion for two dollars? For three dollars? For four dollars? For five dollars? For six dollars?

Suppose there were twelve scholars in your class, how many will half of them be? A quarter? A sixth? Three quarters? Two sixths? Four sixths? Five sixths? A third? Two thirds?

What part of twenty is ten? Five? Fifteen? Nineteen? Four? Eight? Twelve? Sixteen? Two?

If there be forty lines on a page, how many lines will three quarters be?

LESSON NINETEENTH.

If you can buy a sheet of gingerbread for ten cents, you can, of course, buy more than a sheet for fifteen cents; now, how much gingerbread will fifteen cents buy?

If you can buy a sheet of gingerbread for ten cents, you can, of course, buy more than a sheet for twelve cents; now, how much gingerbread will twelve cents buy?

If wood is seven dollars a cord, how much can you buy for eight dollars? For nine dollars? For twelve dollars? For ten dollars? For thirteen dollars? For eleven dollars? For fourteen dollars?

Suppose the school contained twenty-six scholars, and the master should wish to divide it into classes of six each; how many classes could he make out?

If cherries be worth seven cents a quart, how many can you buy for eight cents? How many for ten cents? How many for eleven cents?

How much cloth, at four dollars a yard, can you buy for two dollars? For six dollars?

How much coal at eight dollars per ton, can be purchased for twelve dollars? How much for ten dollars?

How much hay must be sold, at eight dollars a ton, to amount to fourteen dollars?

How many bunches of quills can be bought for one dollar and one fifth of a dollar, if the price of a single bunch is one fifth of a dollar?

If wheat is ten shillings a bushel, how much can you buy for one shilling? What part of a bushel for two shillings? What part for three shillings? What part for four shillings? What part for five shillings? What part for six shillings? What part for seven shillings? What part for eight shillings? How much for nine shillings? How much for eleven shillings? How much for fourteen shillings? How much for nineteen shillings? How much for twenty-one shillings?

LESSON TWENTIETH.

If cherries be worth six cents a quart, how many c... buy for nine? How many can you buy for eight cents? many can you buy for ten cents? How many can you seven cents? How many can you buy for eleven cent...

If strawberries be worth twenty cents a box, what p... box can you buy for one cent? For two cents? ... cents? For five cents? For ten cents? For fifteen... For twenty-two cents? For twenty-four cents? For ... five cents?

In eleven how many times two? How many times th... How many times five? How many times six? How ma... times seven?

In twelve how many times nine? How many times thir... teen? How many times nineteen?

If you pay sixteen cents for twelve pears, how much will they each cost?

How much coffee can you buy for eighteen cents, if it cost twelve cents a pound? How much for twenty-two cents? How much for fifteen cents?

At eleven cents a pound, how much will a half a pound of sugar cost?

If you can buy a barrel of fish for three dollars, how much can you buy for four dollars? For five dollars? For seven dollars? For eight dollars?

Eight are how many times three? How many times three are ten? Eleven? Thirteen?

How many times four are ten? Eleven? Thirteen? Fourteen? Fifteen?

What part of five is two? Three? Seven is how many times five?

Eight are how many times five? How many times five are nine? Eleven? Twelve? Thirteen? Fourteen?

How many times six are eight? Nine? Ten? Eleven? Thirteen? Fifteen? Sixteen? Seventeen?

How many hours will it take you to read through a book of eighty pages, if you read nine pages an hour?

In a school, one sixth of the pupils are studying Welch's Arithmetic; one third are studying the Boston School Atlas; and the rest, which is a class containing twenty scholars, are studying Kelley's Spelling Book: how many pupils were there in the school?

LESSON TWENTY-FIRST.

What part of six is four? How many times six are eight?

Two is what part of seven? How many times seven are nine?

How many times eight are ten? Twelve? Thirteen? Fifteen?

Two thirds and one third make how much?

Eight school boys, at play, broke a globe, valued at fifteen dollars, which they agreed to replace by subscription; what part of the sum must each pay? When the schoolmaster was informed of the act, he said he would be satisfied if the boys would buy one for twelve dollars; how much had each to pay in that case?

If you had twenty-five dollars, how much cloth could you buy at the rate of six dollars a yard? How much at four dollars a yard? How much at three dollars a yard? How much at seven dollars a yard?

At ten cents a pound, how much of an article can be bought for twenty-four cents? For twenty-six cents?

At eleven cents a yard, how much cotton can be bought for twenty-seven cents? How much for twenty-nine cents?

LESSON TWENTY-SECOND.

A dollar is one hundred cents; how many cents make a half dollar? A quarter dollar? Three quarters?

How many cents are there in an eighth of a dollar? In a sixteenth?

Note.—An eighth of a dollar is called, in New England, *nine-pence*; in New York, a *shilling*. A sixteenth is called, in N. E., *four-pence-half-penny*; in N. Y., *six-pence*.

How many cents are there in a dime, or tenth of a dollar? In a half-dime?

Note.—The above moneys are represented by silver coins, except the cent, which is copper. The gold coins of the United States are the eagle (the value of which is ten dollars), the half-eagle, and the quarter-eagle.

If you are to pay a quarter of a dollar for an article, and hand the shopman a dollar, how much ought you to receive in change?

How many eighths of a dollar are there in fifty cents? In thirty-seven and a half cents? In sixty-two and a half cents?

If you pay six and a quarter cents for an orange, and give the shopman a dollar, how many sixteenths ought you to receive back in change?

AMERICAN ARITHMETIC,

AND

PRACTICAL MENSURATOR.

ARITHMETIC is (1) the art of computing by numbers; these numbers are called (2) Figures, viz: (3) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, (4)—0, of itself has no value, but when joined to the right of other numbers, it increases their value in a tenfold proportion. (5) Thus, 1 is one, 10 is ten, 2 is two, 20 is twenty.

There are (6) Five PRINCIPAL or FUNDAMENTAL RULES in Arithmetic, viz:—(7) NOTATION, NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION and DIVISION, and are so called (8) because all mathematical Questions are dependent on them for their operation.

Questions.—1. What is Arithmetic?—2. What are these numbers called?—3. Name them.—4. Describe Zero or Cypher.—5. Give an Example.—6. How many Primary or Fundamental Rules are there in Arithmetic?—7. Name them.—8. Why are they so called?

ON entering upon this most useful study, the first thing which the scholar has to regard, is

(1) *NOTATION.*

NOTATION is (2) the art of expressing numbers by certain characters or figures: of which there are (3) two methods. 1. (4) The *Roman method*, by Letters. 2. (5) The *Arabic method*, by Figures. (6) The latter is that of general use.

In the Arabic method all numbers are expressed (7) by these ten characters or figures, viz.

1 2 3 4 5 6 7 8 9 0

Unit; or one } two; three; four; five; six; seven; eight; nine; cypher or nothing.

The nine first are called (8) *significant figures* or *digits*, each of which standing by itself or alone, invariably expresses (9) a particular or certain number; (10) thus, 1 sig-

nifies *one*, 2 signifies *two*, 3 signifies *three*, and so of the rest, until you come to *nine*.

This will be more particularly taught by

NUMERATION.

Numeration teaches how (11) to *read* or *write* any sum or number by figures.

The value of figures in conjunction, and how to read any sum or number agreeably to the foregoing observations, may be fully understood by the following

TABLE.

1 Millions of Millions.	2 Hundreds of Thou. of Millions.	3 Tens of Thousands of Millions.	4 Thousands of Millions.	5 Hundreds of Millions.	6 Tens of Millions.	7 Millions.	8 Hundreds of Thousands.	9 Tens of Thousands.	4 Thousands.	3 Hundreds.	2 (13) Tens.	3 (12) Units
	2	3	4	5	6	7	8	9	4	3	2	3
	2	3	4	5	6	7	8	9	4	3	2	3
		3	4	5	6	7	8	9	4	3	2	3
			4	5	6	7	8	9	4	3	2	3
				5	6	7	8	9	4	3	2	3
					6	7	8	9	4	3	2	3
						7	8	9	4	3	2	3
							8	9	4	3	2	3
								9	4	3	2	3
									4	3	2	3
										3	2	3
											2	3
												3

Those words at the head of the Table are applicable to any sum or number, and must be committed perfectly to memory so as to be readily applied on any occasion.

The first six figures from the right hand are called the *unit period*, the next six the *million period*, after which the

trillion, quadrillion, quintillion periods, &c. may follow in their order.

Thus by the use of ten figures may be reckoned every thing which can be numbered; things, the multitude of which far exceeds the comprehension of man.

Questions.—1. What is the first thing to be regarded in the study of Arithmetic?—2. What is *Notation*?—3. How many methods of expressing numbers?—4 What is the first?—5. What is the second?—6. Which is generally used?—7. By what are all numbers expressed?—8. What are the nine first figures called?—9. What does each express?—10. Give an Example.—(11) What does *Numeration* teach?—(12) What does the first figure on the right hand of the Table denote?—(13) What the second? Repeat those words at the head of the Table upwards, beginning with Units.

Of other Characters used in Arithmetic.

+ Is the sign of Addition; and shews, that the number which follows the sign, must be added to the number before it. Thus 9+10 signifies that 9 and 10 are to be added.

— Is the sign of Subtraction; and denotes the number following it, must be subtracted from the one before it. Thus, 16—4 signifies that 4 must be taken from 16.

× Is the sign of Multiplication; and denotes that all the numbers, between which it is placed, are to be multiplied together. Thus 9×9 signifies that 9 is to be multiplied by 9, or 9×9×9 must be multiplied.

÷ Is the sign of Division; and denotes the number standing before it, is to be divided by the number following it. Thus, 9÷3 signifies that 9 is to be divided by 3.

= This is the sign of equality; and signifies the sum or product of the numbers before it, is equal to the number after it, 2+4+5=11 mean that 2, 4 and 5 added, their sum would be 11; and 2×4×5=40 mean, that 2 and 4 multiplied, and that product multiplied again by 5, the product is equal to 40; and also 9×8÷12=6, means that 9 and 8 multiplied, and the product divided by 12, the quotient will be equal to 6.

: : : : Are the signs of proportion, as 7: 14: : 8: 16, and are read as 7 to 14, so is 8 to 16.

$\sqrt{^2}$ This shews the Square Root of the number is required.

$\sqrt{^3}$ This shews the Cube Root of the number is required.

1	2	3	4	5	6	7	8	9	10	11	12	13	1
2	4	5	6	7	8	9	10	11	12	13	14	15	2
3	5	6	7	8	9	10	11	12	13	14	15	16	3
4	6	7	8	9	10	11	12	13	14	15	16	17	4
5	7	8	9	10	11	12	13	14	15	16	17	18	5
6	8	9	10	11	12	13	14	15	16	17	18	19	6
7	9	10	11	12	13	14	15	16	17	18	19	20	7
8	10	11	12	13	14	15	16	17	18	19	20	21	8
9	11	12	13	14	15	16	17	18	19	20	21	22	9
10	12	13	14	15	16	17	18	19	20	21	22	23	10

Explanation of the above Table in Addition.

Look for the number to be added on the left or right hand, and the number with which you would add it on the top, and in the corner, or angle in which the lines meet, stands the sum required.—It is required to add 7 with 13; look for 7 on the left, or right, and move along on that line till 13 on the top stands directly over, and in the angle of the two lines stands 20, the sum required.

Explanation of the above Table in Subtraction.

Look for the number to be subtracted on the left or right hand column, and on that line look along towards the right or left hand, till you find the number from which you would subtract, and directly over that number on the top, is a number answering to the remainder.

EXAMPLE.

Required to take 9 from 15, look for 9 on the left or right hand, then follow that line towards the right or left till you find 15, and on the top directly over 15 is 6, the remainder required.

ARITHMETIC.

SIMPLE ADDITION.

Simple addition teaches (1) to add together several sums of the same denomination.

RULE.—Set down (2) units under units, tens under tens, and hundreds under hundreds, &c. Begin to add (3) in the place of units, (4) add all in that column, carry one for every ten that is in the sum of units, to the place of tens, and set down all that is over ten, or tens; (5) add up the column of tens, carry one for every ten to the next column, and set down all that is over, thus proceed through all the columns to the last; and in the last column observe (6) to set down the whole sum.

PROOF.—(7) Begin at the top, add downwards, carry as before, and if the work is right, the sum will be the same as it was when added upwards.

Questions.—1. What does Simple Addition teach?—2. How do you set down sums to be added?—3. Where do you begin to add?—4 How do you add that column?—5. How the columns of tens?—6. What do you observe in the last column?—7. What is the method of Proof?

Examples.

123456	34564	98764321	64321
654321	32567	12345678	4321
123456	37432	82456781	5432
654321	12345	94321452	432
1555554	116908	287888232	74506

1. Add together 345674, 98736, 456752 and 7654.
   Ans. 908816.
2. Add 94321, 675431, 1234 and 76432 together.
   Ans. 847418.
3. Add 6421, 94742, 6752 and 875432 together.
   Ans. 983347.
4. Add 67432, 98891, 65641 and 77421 together.
   Ans. 309385.
5. Add 9944 2343 78764 and 94929 together.
   Ans. 185980.

6. I lent a friend 942 dollars, at one time, at another 741, at another 91; what is the whole sum lent?
Ans. 1774.

7. A man killed 4 hogs, weighing as follows; one 396, one 472, one 510 and one 371; what did they all weigh?
Ans. 1749.

8. I killed an ox, his quarters weighed 642, his hide 105, his tallow 92; what did they all weigh? Ans. 839.

9. A man gave to three sons as follows, to one 3427 dols. to another 3025 dolls. to another 2947 dolls.; what did he give to all of them? Ans. 9399.

10. A man's taxes were as follows, for 3 years; the first 142 dolls. second 165, the third 210 dolls.; what did he pay in three years? Ans. 517.

SIMPLE SUBTRACTION.

Simple Subtraction teaches (1) to take a less number from a greater, that is of the same denomination.

RULE.—Write (2) the largest number down first: and the smallest (3) underneath, units under units, &c. begin to subtract (4) in the place of units; and (5) take the under from the upper figure; set down the remainder: If at any time any particular figure in the lower line is larger than the one immediately over it, (6) subtract the under figure from ten; to the remainder add the upper figure and set it down, and carry one to the next figure in the lower line; thus proceed through the whole.

PROOF.—(7) Add the remainder and lower line together, carry as in Simple Addition, and if the work be right it will be like the upper line.

Questions.—1. What does Simple Subtraction teach?—2. Which number do you set down first?—3. Where the smallest, and in what order?—4. Where do you begin to Subtract?—5. How do you then proceed?—6. What do you do, in case any particular figure in the upper line is smallest?—7. How is Subtraction proved?.

Practical Questions in Subtraction.

1. What is the difference between 24567 and 7456?
Ans. 17111.

2. I lent a friend 2456 dollars and received in part 1342 dollars; what remains due? Ans. 1114 dolls.

3. A merchant owes 34323 dollars, his effects are worth but 22749 dollars; how much will he fall in debt?
Ans. 11574 dolls.

4. A merchant has an estate in his hands valued at 52394 dolls. he owes 31399 dollars; what is he worth after paying his debts? Ans. 20995 dolls.

5. A merchant sent a ship and cargo to sea valued at 11497 dollars and on her return, she and her cargo was valued at 16579 dolls.; what was his gain? Ans. 5082 dolls.

6. A ship builder built a ship, which cost him 6241 dolls. and he sold her for 5927 dolls.; what did he lose?
Ans. 314 dolls.

7. A. borrowed of B. 9427 dolls. and he paid him at one time 4249 dolls. at another time 1427 dolls. and at another time 947 dolls.; what remains due? Ans. 2804 dolls.

SIMPLE MULTIPLICATION.

DEFINITION.—Simple Multiplication is (1) a compendious way of adding; as (2) 9 multiplied by 6 is equal to 54, and is the same as if 9 were set down 6 times and added up.

The number to be multiplied is called (3) the multiplicand.

The number given to multiply by, is called (4) the multiplier.*

The number arising from the operation is called (5) the product.

The following table must be committed to memory before any progress can be made in Multiplication.

Questions.—1. What is Simple Multiplication?—2. Give an example. 3. What do you call the number to be multiplied?—4. The number given to multiply by?—5. The number arising from the operation?—6. What are the multiplicand and multiplier collectively called?

---

* The multiplicand and multiplier are in general terms, called Factors.

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

CASE I.

(1) *When the multiplier is a single figure.*

RULE.—(2) Under the multiplicand write down the multiplier, in the place of units; (3) draw a line underneath, and multiply every figure of the multiplicand, by the multiplier; observing (4) to carry one for every ten to the next product. (5) The whole product of the last figure multiplied, must be set down.

Questions.—1. When is Case 1st applied in Multiplication?—2. What is the first step?—3. What is the 2nd?—4. What are you to observe?—5. What is the last step in this Case?

EXAMPLES.

64324	68459	646246
6	8	9
385944		

PROOF.—(1) Cast the nines from the multiplicand; and all that is over put at the right of a cross, (which is made thus ×); then (2) cast the nines from the multiplier, and all that is over, put at the left of the cross; then (3) cast the nines from the product, and put all that is over at the top of the cross. Then (4) multiply the figure on the left of the cross with the one on the right; cast the 9's from the product, and what is over put at the bottom of the cross,

and (5) if the top and bottom figures are alike, the work is right.

Questions.—1. What is the first step in the method of proof in Multiplication?—2. What is the second?—3. What the third?—4. What the fourth?—5. When is the work known to be right?

NOTE.—The preceding method of proof is liable to one error, viz., if any of the figures in the product should happen to be transposed, as if 468 in the product should happen to be written 648, the work would prove, and an error would still exist.

Second method of Proof.—(1) Divide the product by the multiplier, and if the work is right the quotient will be like the multiplicand.

NOTE.—This method can be used only by those who have attained to a knowledge of division.

Questions.—1. What is the second method of proof?

CASE II.

(1) *When the multiplier consists of several figures.*

RULE.—(2) Set down the multiplier under the multiplicand, units under units, &c. (3) multiply each figure in the multiplier into every figure in the multiplicand, singly, as in case first, observing (4) to set down the first figure of the product exactly under the figure by which you are multiplying; then (5) add the several products together, their sum will be the product required.

Questions.—1. When is Case second in Multiplication applied?—2. What is the first step in this Case?—3. What the second step?—4. What are you to observe?—5. What is the last step in Case second?

EXAMPLES.

	Proof.		*Proof.*
74326	3	12764	0
246	3×4	342	0×2
	3		0
445956		25528	
297304		51056	
148652		38292	
18284196 product.		4365288 product.	

Questions.

No.	*Multiplicands.*		*Multipliers.*		*Products.*
1.	24567	× by	123	=	3021741
2.	12484	× by	482	=	5871488
3.	12545	× by	246	=	3036870
4.	45678	× by	333	=	15210774
5.	23243	× by	2002	=	46532486
6.	34567	× by	4001	=	138302567
7.	23456	× by	1003	=	23526368
8.	90042	× by	9009	=	811138378
9.	701231	× by	1231	=	863215361

CASE III.

(1) *When there are cyphers at the right hand of the multiplicand, or multiplier, or both.*

Rule.—(2) Multiply by the figures only; and to the right of the product annex as many cyphers, as are equal to the number of cyphers in both factors.

Questions.—1. When is Case third in Multiplication applied?—2. What is the general Rule?

EXAMPLES.

	101200	456000	94000
	200	3000	20
Products.	20240000	1368000000	1880000

Questions.

No.	*Multiplicands.*		*Multipliers.*		*Products*
1.	621000	× by	2900	=	1800900000
2.	700000	× by	9900	=	6930000000
3.	854000	× by	7600	=	6490400000
4.	943200	× by	7000	=	6602400000
5.	94944	× by	600	=	56966400
6.	412342	× by	550	=	226788100
7.	671230	× by	123	=	82561290
8.	421233	× by	120	=	50547960
9.	4567852	× by	9000	=	41110668000

Practical Questions in Multiplication.

1. A man had a farm on which he raised 360 bushels of wheat; and he had another, on which he raised 6 times as much; what quantity did he raise on both of them?

Ans. 2520 bushels.

2. A man had an estate which he divided among 9 sons as follows, viz. to the first eight, he gave 333 dollars each;

to the ninth, he gave as much wanting 1000 dollars as to the other eight; I demand the value of the estate, and also the ninth son's share.

Value of the estate 4328 dolls.
The 9th son's share 1664 dolls.

3. A man sheared 364 sheep, six years successively, each sheep neated 3 pounds of wool per year: how much wool had he yearly; and how much in six years?

Ans. Yearly 1092 pounds.
In six years 6552 pounds.

4. I sold 342 tons of iron at 142 dollars per ton: I demand the price of the whole. Ans. 48564 dolls.

5. I sold 742 thousand of boards in the W. I. at 62 dollars per thousand; what did they come to?

Ans. 46004 dolls.

6 What will 6422 quintals of fish come to at 6 dollars per quintal? Ans. 38532 dolls.

7. A man travelled 26 days, and each day he travelled 47 miles, what distance did he travel in the whole time?

Ans. 962 miles.

SIMPLE DIVISION.

DEFINITION.—Simple Division teaches (1) to find how many times a smaller number is contained in a larger.

The number given to be divided, is called (2) the dividend.

The number given to divide by, is called (3) the divisor.

That number which expresses the number of times that the divisor is contained in the dividend, is called (4) the quotient, or answer.

If any thing is left after dividing, it is called (5) the remainder.

Questions.—1. What does Simple Division teach?—2. What is the number to be divided called?—3. The number given to divide by?—4. That number which expresses the number of times that the divisor is contained in the dividend?—5. If any thing is left after dividing, what is it called?

There are five Steps in Division.

Steps.
1st *Write down the Dividend, &c.*
2d *Seek how many times, &c.*
3d *Multiply.*
4th *Subtract.*
5th *Bring down, &c.*

LONG DIVISION.

Long Division is (1) ***when the Divisor is more than twelve.***

CASE 1.

RULE.— (2) Write down the dividend, on the right and left draw a curved line, and on the left hand of the dividend write down the divisor; (3) seek how many times the divisor can be had in a competent number of the first figures of the dividend, and place the figure expressing the number of times at the right of the dividend; (4) multiply the divisor by the same figure, and place the product under the first figures of the dividend, (5) subtract; (6) to the remainder bring down the next figure of the dividend, and divide as before; and thus proceed (7) until all the figures of the dividend are brought down; if at any time when you have brought down a figure the sum is still less than the divisor, (8) a cypher must be placed in the quotient, and another figure must be brought down; the remainder must always be (9) less than the divisor.

PROOF.—(10) Multiply the quotient and divisor together, to the product add the remainder (if there is any) and the sum will be like the dividend, if the work is right.

Questions.—1. When is Division said to be Long?—2. What is the 1st step in Long Division?—3. What is the 2nd?—4. What is the 3d?—5. What is the 4th?—6. What is the 5th?—7. How long do you proceed in this way?—8. When you have brought down a figure and the sum is still less than the divisor, what must be done?—9. How must the remainder always be?—10. What is the method of Proof.

EXAMPLES.

Dividend. *Dividend.*

Div. 16)8721491(232593 *Quo.* *Div.* 24)3102134(129255 *Quo.*

32	24
52	70
48	48
41	222
32	216
94	61
80	48
149	133
144	120
51	134
48	120
3 *Rem.*	14 *Rem.*

Questions.

No.	*Dividends.*		*Divisors.*		*Quotients.*	*Rem.*
1.	1432400	÷	222	=	6452	56
2.	2045600	÷	912	=	2242	896
3.	9400032	÷	203	=	46305	117
4.	9320148	÷	4211	=	2213	1200
5.	7345681	÷	5566	=	1319	4127
6.	5432140	÷	2345	=	2316	1120
7.	6041231	÷	9760	=	618	9551

CASE II.

(1) ***When there are cyphers at the right hand of the divisor.***

RULE.—(2) Cut off the cyphers, and also cut off as many figures, or cyphers from the right of the dividend; (3) divide the remaining part of the dividend as in case first, and (4) to the right of the remainder, place the figures which were cut off from the dividend.

Questions.—1. When do we apply Case 2d in Division?—2. What is the first step?—3. What is the second step?—4. Where do you place those figures which were cut off from the dividend?

EXAMPLES.

1. Divide 5642321 by 432000.

432|000)5642|321(13 *Quo.*
432

1322
1296

26321 *Rem.*

No.	*Dividends.*		*Divisors.*		
2.	764213	÷	12000	=	*Ans.* $63\frac{12}{125}\frac{13}{000}$
3.	943217	÷	5000	=	*Ans.* $188\frac{32}{125}\frac{17}{000}$
4.	674321	÷	11200	=	*Ans.* $60\frac{32}{112}\frac{21}{000}$
5.	432132	÷	9200	=	*Ans.* $46\frac{11}{92}\frac{32}{000}$

NOTE.—To divide by 10, 100, or 1000, &c. is only to cut off as many figures, or cyphers from the right of the dividend as are equal to the number of cyphers on the right of the divisor.

Practical Questions in Division.

1. A man had an estate valued at 2474 dollars and it was

divided equally between twelve sons: I demand the share of each. Ans. $206 $\frac{2}{12}$

2. If a tax of 40000 dollars were to be levied upon 1200 polls; I demand what one poll would pay.

Ans. $33 $\frac{400}{1200}$

3. Divide 1474 dollars equally among 25 men, 30 women, 17 boys, and 27 girls; after taking out 75 dollars for a present to a nephew. Ans. $14 $\frac{13}{99}$

4. Divide 247600 pounds of beef equally among an army of 23724 men. Ans. 10 $\frac{10}{23}\frac{3}{7}\frac{8}{24}\frac{9}{24}$ lbs.

5. Sixty men at a feast, which lasted 3 days, spent 240 dollars per day; how much did each man spend per day, and how much in the whole?

Ans. $4 per day, and $12 the whole.

6. Divide 151200 lbs. of beef equally among an army consisting of 27 regiments, each regiment 7 companies, and each company 100 men. Ans. 8 lbs.

7. If 10500 dollars are given for 750 barrels of flour, I demand the price of one barrel. Ans. $14.

8. If 45 horses were sold in the West Indies for 9900 dollars, I demand the average price. Ans. $220 each.

SHORT DIVISION.

Short Division is (1) ***when the divisor is less than*** 12.

RULE.—(2) Write down the Dividend, and draw a line underneath, and a curve line on the left; and on the left of the curved line place your Divisor. (3) Seek how many times the divisor is contained in a competent number of the figures on the left hand of the dividend, and set the figure expressing that number underneath, and all that is over call so many tens; (4) annex the next figure of the dividend to the tens, divide as at first; thus proceed through the whole, and the quotient will stand (5) under the dividend.

Questions.—1. What is Short Division?—2. What is the 1st step in the Rule of Short Division?—3. What is the 2nd step?—4. What is the 3d?—5. Where will the Quotient stand?

EXAMPLES.

2)423(	3)4567(	4)61234(	5)1234567)
211½ Ans.	1522⅓ Ans.	15308¾ Ans.	246913⅖ Ans.
6)64567(	7)12345(	8)43213(	9)764321(

COMPOUND ADDITION.

DEFINITION.—Compound Addition teaches (1) to add different denominations together; such as dollars, cents and mills; tons, hundreds, quarters, pounds, &c.

RULE.—Write down (2) dollars under dollars, cents under cents, mills under mills; tons under tons, hundreds under hundreds, pounds under pounds, &c. always observing (3) to write every denomination under that of the same name; begin to add (4) in the lowest denomination mentioned; (5) find the sum of that denomination, divide the sum by the number which it takes of this denomination to make one of the next higher, set down the remainder, and carry the quotient to the next denomination; (6) find the sum of the next, and proceed as before through all the denominations, observing (7) to set down the whole sum of the last denomination.

Questions.—1. What does Compound Addition teach?—2. How are the numbers to be placed in order to add?—3. What are you always to observe?—4. Where do you begin to add?—5. How do you then proceed with that denomination?—6. How the next.—7. What are you to observe in the last column?

(6) AMERICAN, OR FEDERAL MONEY.

(1)	10 Mills make	1 Cent.	Marked	*m. c.* or *cts.*
(2)	10 Cents "	1 Dime.	"	*d.*
(3)	10 Dimes "	1 Dollar.	"	*D.* or $.
(4)	100 Cents "	1 Dollar.	"	*D.* or $.
(5)	10 Dollars "	1 Eagle.	"	E.

Questions.—1. How many mills make a cent?—2. How many cents a dime?—3. How many dimes a dollar?—4. How many cents a dollar? 5. How many dollars an eagle?—6. What money is this called?

EXAMPLES.

E.	*D.*	*cts.*	*m.*	*E.*	*d.*	*cts.*	*m.*	*D.*	*cts.*	*m.*
126	8	76	4	1234	4	43	6	4567	79	8
21	4	32	1	321	3	46	5	4347	86	7
2	3	45	6	464	5	43	2	7451	75	6
22	4	32	4	364	6	34	1	3321	99	4
173	0	86	5	2384	9	67	4	19689	41	5*

* Some have objected to having the answers put to these questions, but the author is of opinion that they can do no harm, but will abundantly facilitate the labor of the instructor; if the scholar should attempt to copy without adding up the columns, the book can be taken from him; and if he does copy the two or three first questions, it will be impossible to copy the practical questions.

Practical Questions in Money.

1. Add together $21,71 cts.; $63,42 cts.; $84,96 cts.
Ans. $170,09 cts.

2. A. owes me $91,64 cts, 7 m.; B. $102,75 cts.; and C. $21,42 cts.; what do they all owe me?
Ans. $215,81 cts. 7 m.

3. A. owes B. $91,73 cts.; C. $112,41 cts.; and D. $119, 91 cts.: I demand what he owes to B. C. & D.
Ans. $324,05 cts.

4. A. has a demand upon B. for $642,20 cts.; upon C. for $219; and upon D. $750, 94 cts.; I demand the sum of all his debts. Ans. $1612,14 cts.

5. A farmer sold 3 yoke of oxen; for one yoke he received $105; one $117,75 cts.; and one $95,55 cts.; what did they all come to? Ans. 318,30 cts.

6. What is the value of 3 ships, appraised as follows, viz. one at $4750; one at $6742; and one at $7891?
Ans. $19383.

Lawful Money.

Formerly the money of account of the United States was pounds, shillings, pence and farthings: in the first and second editions of this work, I neglected that method of reckoning altogether, excepting in exchange, and it is my opinion still that the knowledge of lawful money ought to be obliterated entirely from our schools: but the tide of public opinion in many sections of the country is so strong in favor of keeping up that method of reckoning, that I have added a few questions in each rule in that currency for the gratification of those who wish to keep up that method of reckoning; those who do not think it of any consequence, can neglect that part and attend to the Federal Currency altogether.

(4) LAWFUL MONEY.

(1)	4 farthings	make	1 penny,	marked	*qr. d.*
(2)	12 pence	"	1 shilling,	"	*s.*
(3)	20 shillings	"	1 pound,	"	£*.

Questions.—1. How many farthings make a penny?—2. Pence a shilling? 3. Shillings a pound?—4. What money is this called?

* The value of a pound is different in different States, (see exchange,) but in all the New England States it is the same.

EXAMPLES.

£.	s.	d.	£.	s.	d.
123	19	$10\frac{1}{4}$	462	17	$6\frac{1}{2}$
233	12	$4\frac{3}{4}$	143	15	$7\frac{1}{4}$
96	16	$7\frac{1}{2}$	211	11	$6\frac{3}{4}$
12	11	$6\frac{1}{4}$	19	11	$4\frac{1}{2}$
467	0	$4\frac{3}{4}$	837	16	$0\frac{3}{4}$

Practical Questions.

1. A merchant has due from one man, £241 12*s.* $9\frac{1}{2}$*d.* from another £241 17*s.* 6*d.*; from another £211 19*s.* 6*d.*, what is due from them all? Ans. £695 9*s.* $9\frac{1}{2}$*d.*

(4) Troy Weight.

By this Weight is weighed (5) gold, silver, &c.

(1) 24 Grains make 1 Pennyweight, marked *gr. pwt.*
(2) 20 Pennyweights " 1 Ounce, " *oz.*
(3) 12 Ounces " 1 Pound, " *lb.*

Questions—1. How many grains make a pennyweight?—2. Pennyweights an ounce?—3. Ounces a pound?—4. What weight do you call this?—5. What is weighed by this weight?

EXAMPLES

lb.	oz.	pwt	gr.	lb.	oz.	pwt.	gr.
245	10	19	23	2345	11	19	23
146	11	16	20	123	7	8	9
145	2	0	7	34	5	16	17
538	0	17	2	2504	1	5	1

Practical Questions in Troy Weight.

1. A man has two wedges of gold, one weighing 25 lb. 3 oz. 12 pwt., and the other weighing 1 lb. 11 oz. 12 pwt. 7 gr.; I demand the weight of the two wedges.
Ans. 27 lb. 3 oz. 4 pwt. 7 gr.

2. What is the weight of the following silver articles, viz. one spoon weighing 1 lb. 10 oz. 19 pwt.; one tankard weighing 4 lb. 6 oz.; one basin weighing 1 lb. 10 oz. 19 pwt.? Ans. 8 lb. 3 oz. 18 pwt.

(6) Avoirdupois Weight.

By this weight is weighed (1) *sugar, iron, steel, dye woods, and all drossy metals; hay, flour, and many other articles of trade.*

(1)	16 Drams	make	1 Ounce,	marked	*dr.*	*oz.*
(2)	16 Ounces	"	1 Pound,	"		*lb.*
(3)	28 Pounds	"	1 Quarter,	"		*qr.*
(4)	4 Quarters	"	1 Hundred,	"		*cwt.*
(5)	20 Cwt.	"	1 Ton,	"		*ton.*

Questions.—1. How many drams make an ounce?—2. Ounces a pound?—3. Pounds a quarter?—4. Quarters a hundred?—5. Hundreds a ton?—6. What weight is this called?—7. What is weighed by this weight?

EXAMPLES.

Tons.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*	*Tons.*	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
444	12	1	12	2	13	123	12	2	22	10	12
41	11	0	10	12	13	321	19	3	27	15	15
623	19	3	12	15	15	141	12	2	19	11	12
610	3	1	7	15	9	587	5	1	14	6	7

Practical Questions in Avoirdupois Weight.

1. Add together the following parcels, viz. 12 cwt. 2 qr. 17 lb.; 1 ton, 19 cwt. 3 qr. 27 lb. 15 oz. 15 dr.; and 2 tons, 12 cwt. 2 qr. 20 lb. 12 oz. 10 dr.

Ans. 5 tons, 5 cwt. 1 qr. 9 lb. 12 oz. 9 dr.

2. What is the weight of 3 hhds. of sugar, weighing as follows, viz. 1st, 9 cwt. 3 qr. 12 lb.; the 2d, 10 cwt. 1 qr. and 12 lb.; and the 3d, 8 cwt. 3 qr. 27 lb.?

Ans. 29 cwt. 0 qr. 23 lb.

(5) Apothecaries' Weight.

By this weight is weighed (6) *Medicine, &c.*

(1)	20 Grains	make	1 Scruple,	marked	*gr.*	*sc.*
(2)	3 Scruples	"	1 Dram,	"		*dr.*
(3)	8 Drams	"	1 Ounce,	"		*oz.*
(4)	12 Ounces	"	1 Pound,	"		*lb.*

Questions.—1. How many grains make a scruple?—2. Scruples a dram?—3. Drams an ounce?—4. Ounces a pound?—5. What weight is this called?—6 What is weighed by this weight?

EXAMPLES.

lb.	oz.	dr.	sc.	gr.	lb.	oz.	dr.	sc.	gr.
23	6	7	2	19	121	11	7	2	19
10	5	4	1	10	911	2	6	1	11
11	0	3	2	11	191	1	4	1	9
45	1	0	1	0	1224	4	2	2	19

Practical Questions in Apothecaries' Weight.

1. Add together 22 lb. 0 oz. 1 dr. 2 sc. 13 gr.; 12 lb. 0 oz. 6 dr. 2 sc. 19 gr.; and 22 lb. 1 dr. 1 sc. 19 gr.

Ans. 56 lb. 1 oz. 2 dr. 1 sc. 11 gr.

2. Add together 11 lb. 11 oz. 6 dr. 2 sc. 12 gr.; 21 lb. 6 oz. 6 dr.; and 122 lb. 9 oz. 2 dr. 1 sc. 10 gr.

Ans. 156 lb. 3 oz. 7 dr. 1 sc. 2 gr.

(12) LONG MEASURE.

By this measure is measured (13) *distances; and by some it is called running measure.*

(1)	3 Barley Corns	make	1 Inch, marked	*bar*	*in.*
(2)	12 Inches	"	1 Foot, "		*ft.*
(3)	3 Feet	"	1 Yard, "		*yd.*
(4)	5½ Yards	"	1 Rod, "		*rd.*
(5)	16½ Feet	"	1 Rod, "		*rd.*
(6)	40 Rods	"	1 Furlong, "		*fur.*
(7)	8 Furlongs	"	1 Mile, "		*M.*
(8)	3 Miles	"	1 League, "		*lea.*
(9)	60 Geographical or	}	1 Degree of the Earth's		
(10)	69¼ Statute Miles	}	great circle,		*deg.*
(11)	360 Degrees	"	1 Circle, "		*cir.*

Questions.—1. How many barley corns make an inch?—2. Inches a foot?—3. Feet a yard?—4. Yards a rod?—5. Feet a rod?—6. Rods a furlong?—7. Furlongs a mile?—8. Miles a league?—9. Geographical miles a degree?—10. Statute miles a degree?—11. Degrees a circle?—12. What measure is this called?—13. What is measured by this measure?

EXAMPLES

Deg.	*St.m.*	*fur.*	*rd.*	*ft.*	*in.*	*bar.*	*Miles.*	*fur.*	*rd.*	*yd.*	*ft.*
122	68	6	30	15	10	2	240	7	39	4	2
211	10	4	20	15	9	1	213	6	11	2	1
31	6	2	39	14	2	2	12	4	31	2	2
21	37½	3	4	12	1	2	144	2	22	4	1
386	53½	1	16	8½	0	1	611	5	25	3	0

Practical Questions in Long Measure.

1. The distance from A. to B. is 3 miles 6 fur. 27 rods; from B. to C. is 1 lea. 2 miles, 7 fur.; from C. to D. 27 lea. 1 mile, 2 fur. 39 rods; I demand the distance from A. to D.

Ans. 30 lea. 2 miles 0 fur. 26 rd.

2. A ship sailed south 6 lea. 1 mile; S. S. E. 9 lea. 2 miles 0 fur. 39 rd.; E. S. E. 21 lea. 1 mile 6 fur. 7 rds.; what was the whole distance sailed?

Ans. 37 lea. 1 mile, 7 fur. 6 rds.

( ) Land or Square Measure.

This measure is used in measuring (5) *all things that have length and breadth without regard to thickness.*

(1)	144	Inches	make	1	Foot,	marked	*in. ft.*
(2)	9	Feet	"	1	Yard,	"	*yd.*
(3)	30¼	Yards	"	1	Rod,	"	*rd.*
(4)	272¼	Feet	"	1	Rod,	"	*rd.*
(5)	40	Rods	"	1	Rood,	"	*rood.*
(6)	4	Roods	"	1	Acre,	"	*acre.*
(7)	160	Rods	"	1	Acre,	"	*acre.*

Questions.—1. How many inches make a square foot?—2. Feet a yard?—3. Yards a rod?—4. Feet a rod?—5. Rods a rood?—6. Roods an acre?—7. Rods an acre?—8. What measure is this called?—9. What are measured by it?

EXAMPLES.

Acres.	*roods.*	*rds.*	*ft.*	*in.*	*Acres.*	*roods.*	*rds.*
272	3	39	216	123	212	2	29
142	2	26	270	143	143	2	29
128	1	10	16	43	143	2	10
321	0	7	8	139	341	3	39
860	0	3	240¾	16	846	3	27

Practical Questions in Land, or Square Measure.

1. A man had several lots of land; the 1st contained 27 acres 3 roods; the 2d, 19 acres, 2 roods, 27 rods, 17 feet; and the 3d, 6 acres, 2 roods, 39 rods: I demand the quantity in the 3 lots. Ans. 54 acres, 26 rods, 17 feet.

2. Add together 64 acres, 2 roods, 39 rods; 61¼ acres, 3 roods, 17 rods; and 104 acres, 2 roods, 22 rods.

Ans. 231 acres, 0 roods, 38 rods.

(11) Wine Measure.

By this measure, are measured (12) *wine, rum, gin, and almost every thing that is sold by the gallon, &c. A gallon wine measure is equal to 231 solid, or cubic inches.*

(1) 4 Gills	make	1 Pint,	marked	*gil. pt.*
(2) 2 Pints	"	1 Quart,	"	*qt.*
(3) 4 Quarts	"	1 Gallon,	"	*gal.*
(4) 42 Gallons	"	1 Tierce,	"	*tier.*
(5) 63 Gallons	"	1 Hogshead,	"	*hhd.*
(6) 84 Gallons	"	1 Puncheon,	"	*pun.*
(7) 126 Gallons or (8) 2 hhds.	"	1 Pipe,	"	*pipe.*
(9) 2 Pipes, or (10) 252 gal.		1 Tun,	"	*tun.*

Questions.—1. How many gills make a pint?—2. Pints a quart?—3. Quarts a gallon?—4. Gallons a tierce?—5. Gallons a hogshead?—6. Gallons a puncheon?—7. Gallons a pipe?—8. Hogsheads a pipe?—9. Pipes a tun?—10. Gallons a tun?—11. What measure is this called?—12. What are measured by it?

EXAMPLES.

Pun.	*gal.*	*qt.*	*pt.*	*Pipes.*	*hhd.*	*gal.*	*qt.*	*pt.*
143212	38	3	1	123	1	62	3	1
43200	71	1	0	43	1	41	2	0
32111	21	2	1	4	0	21	1	1
272	10	1	0	3	1	17	3	0
218796	58	0	0	175	1	17	2	0

Practical questions in Wine Measure.

1. What is the sum of the following parcels, viz. 19 gal 2 qt. 1 pt.; 12 gal. 1 qt.; 21 gal. 3 qt.; and 1 hhd. 3 gal.?

Ans. 1 hhd. 56 gal. 2 qt. 1 pt.

2. A merchant received from Lisbon at one time 123 pip. 1 hhd. 62 gal. 3 qt. 1 pt.; at another time 43 pip. 1 hhd 41 gal. 2 qt.; and at another time he received 4 pip. 40 gal. what did he receive in all?

Ans. 172 pip. 18 gal. 1 qt. 1 pt.

(4) ALE AND BEER MEASURE.

By this measure is bought and sold (3)*beer, milk, &c. One ale or beer gallon contains 282 solid inches.*

(1)	2 Pints	make	1 Quart,	marked	*pt. qt.*
(2)	4 Quarts	"	1 Gallon,	"	*gal.*
(3)	8 Gallons	"	1 Firkin Ale,	"	*F. fir.*
(4)	9 Gallons	"	1 Firkin Beer,	"	*B. fir.*
(5)	36 Gallons	"	1 Barrel,	"	*bbr.*
(6)	1½bbr.or 54G.	"	1 hhd.,	"	*hhd.*
(7)	3 Barrels	"	1 Butt,	"	*butt.*

Questions.—1. How many pints make a quart?—2. Quarts a gallon?—3. Gallons a firkin of ale?—4. Gallons a firkin of beer?—5. Gallons a barrel?—6. Barrels or gallons a hogshead?—7. Barrels a butt?—8. What measure is this called?—9. What are measured by it?

EXAMPLES.

Hhd.	*gal.*	*qt.*	*pt.*	*Bbr.*	*gal.*	*qt.*	*pt.*
22	51	3	1	122	31	3	1
91	52	2	1	920	33	3	1
9	43	1	0	91	15	1	1
6	41	2	1	9	29	2	1
131	27	1	1	1145	2	3	0

Practical Questions in Ale and Beer Measure.

1. Add together the following quantities of beer, viz.—33 hhds. 51 gal. 2 qts.; 4 hhds. 52 gal. 2 qts.; and 19 hhds. 16 gal. 1 qt. Ans. 58 hhd. 12 gal. 1 qt.

2. The following parcels of beer were sold to a merchant: first, 19 hhd. 27 gal. 3 qts.; second, 31 hhd. 21 gal. 2 qts.; at the third time, 11 hhd. 6 gal. 3 qts.: what was sold him at the three times? Ans. 62 hhd. 2 gal. 0 qt.

SOLID MEASURE.

By this is measured (5)*all such things as have length, breadth and thickness; such as stone, timber, wood, bark, &c.*

(1)	1728 Inches	make	1 Foot,	marked	*in. ft.*
(2)	27 Feet	"	1 Yard,	"	*yd*
(3) (4)	40 Feet of round, or 50 Feet of hewn timber		1 Ton, or Load,		*ton.*

Questions.—1. How many inches make a foot in Solid Measure?—2. Feet a yard?—3. Feet of round timber a ton?—4. Feet of Hewn timber a ton?—5. What are measured by this measure?

EXAMPLES.

H. tons.	*ft.*	*in.*	*Yds.*	*ft.*	*in*
222	49	1721	234.	26	1727
321	36	246	678	21	1672
26	41	1000	565	20	1242
9	36	1259	69	21	43
581	14	770	1549	9	1228

Practical Questions.

1. What is the sum of the three following sticks of timber, viz. the first, 3 tons, 49 ft. 1672 inches; the second, 4 tons 47 ft. and 1000 in.; the third, 6 tons, 40 ft. 1001 in.?
Ans. 15 tons, 39 ft. 217 in.

2. Add together the following parcels, viz. 29 tons, 30 ft.; 56 tons, 39 ft. 127 in.; 61 tons, 19 ft.; and 71 tons, 27 ft. 1649 in. Ans. 219 tons, 36 ft. 48 in.

(5) Wood and Bark Measure.

Wood and bark are bought and sold by the cord or foot; a foot of wood or bark contains 16 *solid feet, or one eighth of a cord; and a pile* (6) 4 *feet high,* 4 *feet wide and* 8 *feet long contains one cord.*

(1)	1728 Inches make	1 Solid foot, marked	*in.*	*ft.*
(2)	16 Solid feet "	1 Foot of wood, "		*ft.*
(3)	8 Feet of wood	1 Cord, "		*cord.*
(4)	128 Solid feet "	1 Cord, "		*cord.*

Questions.—1. How many inches make a solid foot?—2. How many solid feet a foot of wood?—3. How many feet of wood a cord?—4. How many solid feet a cord?—5. What measure is this called?—6. What must be the dimensions of a pile of wood to contain a cord?

EXAMPLES

cords.	*ft.*	*in.*	*cords.*	*ft.*	*in.*
9	127	1642	64	90	146
2	106	642	9	10	140
40	50	60	70	80	90
61	11	9	10	21	31
114	33	625	154	73	407

Practical Questions in Wood and Bark Measure.

1. Add together the following parcels of wood, viz. 64 cords, 120 feet; 72 cords, 21 feet; and 122 cords.

Ans. 259 cords, 13 s. feet.

2. Add 3 cords 4 feet of wood, 12 solid feet; 6 cords 5 feet of wood, and 12 solid feet; 9 cords and 6 feet of wood together. Ans. 20 cords, 8 s. feet.

DRY MEASURE.

By this measure is bought and sold (8) *salt, corn, and all kinds of grain; 268.8 solid or cubic inches make one gallon of corn, &c.*

(1)	2 Pints	make	1 Quart,	marked	*qt. pt.*
(2)	4 Quarts	"	1 Gallon,	"	*gal.*
(3)	2 Gallons	"	1 Peck,	"	*pk.*
(4)	8 Quarts	"	1 Peck,	"	*pk.*
(5)	4 Pecks or 32 qts.		1 Bushel,	"	*bu.*
(6)	8 Bushels	"	1 Quarter,	"	*qr.*
(7)	36 Bushels	"	1 Chaldron,	"	*ch.*

Questions.—1. How many pints in dry measure make a quart?—2. How many quarts a gallon?—3. Gallons a peck?—4. Quarts a peck?—5. Pecks or quarts a bushel?—6. Bushels a quarter?—7. Bushels a chaldron?—8. What are bought and sold by this measure?

EXAMPLES

Ch.	*bu.*	*pks.*	*gal.*	*qts.*	*pts.*	*Qr.*	*bu.*	*pks.*	*gal.*	*qts.*	*pts.*
654242	32	2	1	2	0	42	7	3	1	3	1
52312	31	0	0	3	1	312	6	2	0	2	0
213	20	3	1	2	0	743	2	1	1	1	0
4201	11	2	0	1	1	7	6	3	0	3	1
710970	24	1	0	1	0	1106	7	3	0	2	0

Practical Questions in Dry Measure.

1. Add together the following parcels, viz. 1627 bu. 3 pks. 7 qts. 1 pt.; 972 bu. 2 pks. 1 qt.; 2471 bu. 1 pk. 2 qts. 1 pt. Ans. 5071 bu. 3 pks. 3 qts.

2. The following quantities of coal were sold to a Merchant, viz. 41 chal. 30 bu. 3 pks.; 51 chal. 35 bu. 1 pk.; and 60 chal. 7 bu.; what was the quantity sold?

Ans. 154 chal. 1 bu. 0 pk.

Cloth Measure.

By this measure is bought and sold (7) *cloths and silks, &c.*

(1)	2½	Inches	make	1	Nail,	marked	*in. na.*
(2)	4	Nails	"	1	Quarter,	"	*qr.*
(3)	4	Quarters	"	1	Yard,	"	*yd.*
(4)	3	Quarters	"	1	Ell Flemish,	"	*E. Fl.*
(5)	5	Quarters	"	1	Ell English,	"	*E. E.*
(6)	6	Quarters	"	1	Ell French,	"	*E. F.*

Questions.—1. How many inches make a nail in cloth measure?—2. How many nails a quarter?—3. Quarters a yard?—4. Quarters an ell Flemish? 5. Quarters an ell English?—6. Quarters an ell French?—7. What are bought and sold by this measure?

EXAMPLES.

F.E.	*qr.*	*na.*	*E.E.*	*qr.*	*na.*	*E.Fl.*	*qr.*	*na.*	*Yds.*	*qr.*	*na.*
6543	5	1	244	4	3	246	2	2	126	3	3
3454	4	8	321	3	1	234	0	2	245	3	3
6543	3	2	121	1	2	121	2	1	213	1	0
6543	2	1	200	3	3	303	2	1	123	1	3
23085	3	3	879	3	1	906	1	2	709	2	1

Practical Questions in Cloth Measure.

1. Add together the following pieces of cloth, viz. 29 yds. 3 qr. 2 na.; 30 yds. 1 qr. 3 na.; 36 yds. 1 qr. 1 na.; and another piece containing as much as all the others.

Ans. 193 yds. 1 qr.

2. How many French ells in the following pieces of cloth, viz. first 22 qr. 3 na.; second 31 qr. 1 na.; third 50 qr. 2 na.; and fourth 91 qr. 1 na.? Ans. 32 F. ells, 3 qr. 3 na.

Of Time.

Time is divided into (1) *seconds, minutes, hours, days, weeks, months and years.*

(2)	60	Seconds	make	1	Minute,	marked	*se. m.*
(3)	60	Minutes	"	1	Hour,	"	*h.*
(4)	24	Hours	"	1	Day,	"	*d.*
(5)	7	Days	"	1	Week,	"	*w.*
(6)	4	Weeks	"	1	Month,	"	*mo.*
(7)	13	Months	"	1	Lunar year,	"	*year.*
(8)	12	Solar months	"	1	Year,	"	*year*

(9) 365 Days, 5 hours, 48 min. 57 se. make 1 Solar year.

To remember the number of days in each calendar month, commit to memory (10) the following lines:—viz.

Thirty days hath September,
April, June and November;
All the rest have thirty one
Except the second month alone,
Which hath but twenty-eight in fine
'Till leap year gives it twenty nine.

NOTE.—(11) *When any year can be divided by 4 without a remainder, it is leap year; and February has 29 days.*

Questions.—1. What are the divisions of time?—2. How many seconds make a minute?—3. Minutes an hour?—4. Hours a day?—5. Days a week?—6. Weeks a month?—7. Months a lunar year?—8. Months a solar year?—9. How many days, &c. make a solar year?—10. What should be committed to memory in order the more perfectly to remember the number of days in each calendar month?—11. When is any year considered Bissextile or leap year?

EXAMPLES.

L.yrs.	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
123	11	0	6	23	59	59
123	11	1	4	21	20	40
191	12	3	2	11	12	10
251	11	1	2	10	10	9
691	7	3	2	13	42	53

L.yrs.	*mo.*	*d.*	*h.*	*m.*	*s.*
12	12	27	23	59	59
19	9	4	11	12	11
21	4	6	1	10	9
41	2	4	6	9	40
95	2	14	18	31	59

Practical Questions in Time.

1. Add together 12 years, 6 mo. 3 w. 6 d. 23 h. 12 m. 12 s.; 49 years, 5 mo. 2 w. 4 d. 12. h. 9 m.; 1 year, 9 mo. 1 w. 1 d. 1 h. 6 m.

Ans. 63 yrs. 8 mo. 3 w. 5 d. 12 h. 27 m. 12 s.

2. Add 21 yrs. 9 mo. 6 d.; 31 yrs. 6 mo. 21 d.; 6 yrs. 4 mo. 9 d.; and 12 yrs. 7 mo. 10 d. together.

Ans. 72 yrs. 1 mo. 18 d.

OF MOTION.

(1) 60 Seconds make 1 Prime Minute, marked ″ ′
( ) 60 Minutes " 1 Degree, " °
(2) 30 Degrees " 1 Sign, " *s.*
(3) 12 Signs, or 360 Degrees the whole great circle of the [Zodiac.

Questions.—1. How many seconds a prime minute?—2. How many

minutes a degree?—3. Degrees a sign?—4. Signs or degrees a circle of the Zodiac?

EXAMPLES.

°	′	″
25	17	18
17	49	56
6	35	24
10	17	16

s.	°	′	″
9	8	55	44
1	26	44	55
8	18	36	12
1	9	33	22

The following are denominations of things counted by the Tale.

(1) 12 Particular things make 1 Dozen,
(2) 12 Dozen " 1 Gross,
(3) 12 Gross or 144 doz. " 1 Great Gross.

ALSO,

(4) 20 Particular things make 1 Score.

Questions.—1. How many particulars make a dozen?—2. How many doz. a gross?—3. Gross or doz. a great gross?—4. Particulars a score?

Denominations of Measures not included in the Tables.

(1) 6 Points make 1 Line,
(2) 12 Lines " 1 Inch,
(3) 4 Inches " 1 Hand
(4) 3 Hands " 1 Foot,
(5) 66 Feet, or 4 Poles, a Gunter's Chain,
(6) 3 Miles " 1 League.

A Hand is used (7) to measure horses. A Fathom (8) to measure depths. A League (9) in reckoning distances at sea.

N. B. A Quintal of Fish weighs (10) 1 cwt. Avoirdupois.

Questions.—1. How many points make a line?—2. How many lines an inch?—3. Inches a hand?—4. Hands a foot?—5. Feet or poles a gunter's chain?—6. Miles a league?—7. For what is a hand used?—8. A fathom?—9. A league?—10. What is the weight of a quintal fish?

NOTE.—The pupil, after answering the preceding questions, both in simple and compound numbers, should as a method of review, be required to answer them promiscuously, either by himself or in a class; and should any difficulty be found in giving an answer, he is advised to turn back and consult his rule, until a thorough knowledge of the same is obtained.

COMPOUND SUBTRACTION.

DEFINITION.—Compound Substraction teaches (1) to find the difference between a larger and a smaller number, which have several denominations.

RULE.—Write down (2) the largest of the two numbers first, and under it write down the smallest; observing (3) to place every denomination under that of the same name, dollars under dollars, &c.; begin to subtract (4) in the lowest denomination, and take the lower from the upper line; if any particular figure in the lower line in any of the denominations, is larger than the one over it, (5) subtract the lower figure from the number that it takes of this to make one of the next higher denomination; and to the remainder add the top figure, and set the sum down, and carry one to the next denomination; proceed thus through all the denominations.

PROOF.—(6) Add together the remainder and lower line, carry as in compound addition; the sum will be like the upper line if the work is right.

Questions.—1. What does compound subtraction teach?—2. How are the two numbers for subtraction wrote down?—3. What are you to observe?—4. Where do you begin to subtract?—5. If any particular figure in the lower line in any of the denominations, is larger than the one over it, how do you then subtract?—6. What is the method of proof?

AMERICAN MONEY.*

Dolls.	*cts.*	*m.*	*Dolls.*	*cts.*	*m.*	*Dolls.*	*cts.*	*m.*
1241	79	9	4312	29	7	1000	00	0
992	87	8	704	21	6	999	99	9
248	92	1	3608	8	1			1

Practical Questions in American Money.

1. A man lent 3 E. 3 dolls. 3 d. 3 c.; and received 2 E. 2 dolls. 99 cts. and 9 mills; How much remains due?

Ans. 10 dolls. 33 cts. 1 m.

* Scholars cannot be too frequently and thoroughly exercised in the several tables of weights, measures, &c. while passing through subtraction as well as addition; for this is certain, that no scholar can be too familiar with compound numbers; as their use will frequently occur in subsequent rules and examples of arithmetic.

2. A man had an estate valued at 3505 dolls. 6 d. 3 cts. 2 m.; he had three sons, to each of whom he gave 504 dolls. 5 cts. 2 m.; he also gave to two daughters, 75 dolls. 75 cts. each; what had he left? Ans. 1841 dolls. 97 cts. 6 m.

3. A man was worth 9472 dolls. 65 cts.; by a misfortune at sea, he lost 4549 dolls. 75 cts.; What was he worth after he sustained the loss? Ans. 4922 dolls. 90 cts.

LAWFUL MONEY.

£.	s.	d.	£.	s.	d.
647	16	6¼	912	11	4½
549	17	7½	897	12	6¾
97	18	10¾	14	18	9¾

Practical Questions in Lawful Money.

1. What is the difference between £141 17*s.* 6*d.* and £241 12*s.* 9½*d.*? Ans. £99 15*s.* 3½*d.*

2. Lent £91 12*s.* 9½*d.* and received £30 10*s.* 11½*d.* what remains due? Ans. £1. 1*s.* 9¾*d.*

TROY WEIGHT.

lb.	*oz.*	*pwt.*	*grs.*	*lb.*	*oz.*	*pwt.*	*grs*
223	10	19	22	81234567	6	12	20
124	11	12	23	13432148	10	13	22
98	11	6	23	17802418	7	18	22

Practical Questions in Troy Weight.

1. A person had a basin which weighed 3 lb. 10 oz.; it being melted in part, it then was weighed again, and its weight was 2 lb. 11 oz. 7 pwt. 4 grs.; what was melted off?
Ans. 10 oz. 12 pwt. 20 grs.

2. A silver smith had 21 lb. 9 oz. of silver; he refined it by melting, and then weighed it, and its weight was 15 lb. 10 oz. 11 pwt. 19 grs.; what was lost by melting?
Ans. 5 lb. 10 oz. 8 pwt. 5 grs.

8. How much silver must be given in exchanging a silver plate for a platter; the weight of the platter being 2 lb. 10 oz.; and the weight of the plate 1 lb. 4 oz. 7 pwt.?

Ans. 1 lb. 5 oz. 13 pwt.

AVOIRDUPOIS WEIGHT.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*	*T.*	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
12	19	2	27	14	14	134	12	3	20	13	12
9	18	3	24	15	15	94	19	2	21	14	15
3	0	3	2	14	15	39	13	0	26	14	13

Practical Questions in Avoirdupois Weight.

1. A certain vessel was freighted with 22 tons, 13 cwt. 1 qr. of iron; in a storm she was lighted by throwing a part of the cargo overboard; on her arrival at port, her cargo weighed but 15 tons, 0 cwt. 1 qr.; I demand the quantity lost? Ans. 7 T. 13 cwt.

2. A merchant had 31 cwt. 2 qr. 21 lb. of sugar; he sold to A. 11 cwt. 1 qr.; and to B. 15 cwt. 2 qr. 17 lb.; what had he left? Ans. 4 cwt. 3 qr. 4 lb.

3. A merchant had 1 T. 3 qr. 27 lb. of sugar, it was all stolen but 1 cwt. 1 qr. 8 lb.; I demand the quantity stolen? Ans. 19 cwt. 2 qr. 19 lb.

4. I bought 27 cwt. 2 qr. 19 lb. of flour, and sold 19 cwt. 3 qr. 27 lb.; what remained on hand?

Ans. 7 cwt. 2 qr. 20 lb.

APOTHECARIES' WEIGHT.

lb.	*oz.*	*dr.*	*sc.*	*grs.*	*lb.*	*oz.*	*dr.*	*sc*	*grs.*
14	10	6	2	19	123	6	1	1	19
6	11	7	1	16	94	11	6	2	17
7	10	7	1	3	28	6	2	2	2

Practical Questions.

1. An apothecary had 12 lb. 6 oz. 2 dr. of medicine in a certain vessel; by accident it was broken, and there remained in a broken piece of it, 8 lb. 11 oz. 7 dr. 1 sc. 13 grs.; I demand the quantity lost?

Ans. 3 lb. 6 oz. 2 dr. 1 sc. 7 gr.

2. What remains after taking 19 lb. 6 oz. 3 dr. from 26 lb. 7 oz. 7 dr.? Ans. 7 lb. 1 oz. 4 dr.

3. What is the difference between 119 lb. 6 oz., and 111 lb. 11 oz.? Ans. 7 lb. 7 oz.

LONG MEASURE.

Deg.	*mi.*	*fur.*	*rds.*	*ft.*	*in.*	*bar.*	*Mi.*	*fur.*	*rds.*	*ft.*	*in.*	*bar.*
27	59	6	30	15	11	2	224	3	21	13	1	1
9	27	7	35	13	9	2	95	6	32	14	9	2
18	31	6	35	2	2	0	128	4	28	14½	3	2

Practical Questions in Long Measure.

1. Subtract 24 mi. 3 fur. 26 rds. from 26 mi. and shew the difference. Ans. 1 mi. 4 fur. 14 rds.

2. Express the difference between 200 mi. 6 fur. 29 rds.; and 150 mi. 7 fur. 2 rds. 4 yds. 2 ft. 1 bar.

Ans 49 mi. 7 fur. 26 rds. 0 yds. 0 ft. 11 in. 2 bar.

3. Express the difference between 7 lea. 2 mi. 6 fur. 27 rds.; and 5 lea. 1 mi. 7 fur. 30 rds.

Ans. 2 lea. 0 mi. 6 fur. 37 rds.

4. A man agreed with another to go 51 mi. 4 fur., and he went 60 mi. 2 fur.; how much further did he go than he agreed to? Ans. 8 mi. 6 fur.

LAND, OR SQUARE MEASURE.

Acres	*roods*	*rods*	*feet*	*Acres*	*roods*	*rods*	*feet*	*inches.*
126	1	39	16	2456	1	12	10	11
112	3	30	14	1367	2	22	9	7
13	2	9	2	1088	2	30	1	4

Practical Questions in Square Measure.

1. A man's farm contained 246 acres, 1 rood, 7 rods; he sold to A. 27 acres, 2 roods, 1 rod; to B. 39 acres, 39 rods; and to C. 50 acres, 2 roods; how much had he left?

Ans. 129 acres, 7 rods.

2. A man had 700 acres of land; and he had four sons; to the oldest he gave 60 acres 2 roods; and to the second, third and fourth, he gave half as much to each as he did to the first; how much had he left? Ans. 548 acres, roods.

3. A man had 3472 feet 142 inches of boards, he sold 1249 feet 126 inches; what had he left?

Ans. 2223 feet, 16 inhces.

WINE MEASURE.

Punn.	*gal.*	*qts.*	*pt.*	*Pip.*	*hhd.*	*gal.*	*qts.*	*pt.*
12345	70	3	1	23	1	60	2	1
10947	75	2	0	19	0	59	3	0
1397	79	1	1	4	1	0	3	3

Practical Questions in Wine Measure.

1. A wine seller bought 22 hhds. 54 gals. of wine; he sold 12 hhds. 60 gals.; I demand the quantity unsold?

Ans. 9 hhds. 57 gals.

2. A. bought 247 gals. 3 qts. 1 pt. of rum; and he sold from it 198 gals. 2 qts.; how much had he left?

Ans. 49 gals. 1 qt. 1 pt.

3. I demand the difference between 3 pipes, 1 hhd. 54 gals.; and 1 pipe, 1 hhd. 61 gals.

Ans. 1 pipe, 1 hhd. 56 gals.

SOLID MEASURE.

H. tons.	*ft.*	*in.*	*H. tons.*	*ft.*	*in.*
246	49	1720	12	16	1111
179	46	1723	9	49	1727
67	2	1725	2	16	1112

Practical Questions in Solid Measure.

1. A man had 249 tons 12 feet of round timber lying in the dock; in a high tide a part of it went adrift; there remained in the dock 142 tons, 12 feet, 1721 inches; I demand the quantity lost. Ans. 106 tons, 39 feet 7 in.

2. What is the difference between 49 tons, 39 feet; and 22 tons, 38 feet, 1272 inches?

Ans. 22 tons, 0 feet, 456 inches.

3. A stone cutter had on hand 1427 feet of stone; he sold 192 feet, 1427 inches; I demand the quantity unsold.

Ans. 1234 feet, 301 inches.

WOOD MEASURE.

cords.	*sd.ft.*	*cords.*	*sd.ft.*	*cords.*	*sd.ft.*
1846	127	1327	120	2456	11
1297	121	1319	101	1341	125
49	6	8	19	1114	14

Practical Questions in Wood Measure.

1. A wood seller bought 127 cords, 120 solid feet of wood, and sold 100 cords, 101 solid feet; how much had he left on hand? Ans. 27 cords, 19 feet.

2. A coaster freighted his vessel with 65 cords, 118 solid feet of wood; in a storm he was obliged to light her by throwing wood overboard; when she arrived in port, there was but 21 cords, 106 solid feet on board; I demand the quantity lost? Ans. 44 cords, 12 feet.

3. What is the difference between 27 cords, 104 feet, and 19 cords, 125 feet? Ans. 7 cords, 107 feet.

DRY MEASURE.

chal.	*bu.*	*pks.*	*gal.*	*qts.*	*pt.*	*chal.*	*bu.*	*pks.*	*qts.*	*pt.*
224	30	2	1	3	1	3456	27	3	6	0
19	35	3	0	2	0	1967	30	2	7	1
204	30	3	1	1	1	1483	33	0	6	1

Practical Questions in Dry Measure.

1. What is the difference between 7953 chal. 13 bu.; and 3789 chal. 35 bu,? Ans. 4163 chal. 14 bu.

2. A merchant in Boston had a vessel coming from Virginia freighted with 2478 bushels, 3 pecks of corn; in a storm she was lighted by shovelling corn overboard, on her arrival in port, she had but 2027 bushels, 1 peck on board; I demand the quantity lost. Ans. 451 bu. 2 pks.

CLOTH MEASURE.

Fr.E.	*qr.*	*na.*	*En.E.*	*qr.*	*na.*	*Yds.*	*qr.*	*na.*
12	4	3	21	4	3	121	3	2
9	5	3	19	2	3	109	2	3
2	5	0	2	1	0	12	0	3

Practical Questions in Cloth Measure.

1. A dealer in cloths had 3 pieces containing in all 80 yards, 3 qrs.; he sold 51 yards, 3 qrs. 3 nails; I demand what he had left? Ans. 28 yds. 3 qrs. 1 na.

2. A merchant bought 6 pieces of cloth, each piece containing 26 yds. 3 qr. 3 na., he sold two pieces; how many yards had he left? Ans. 107 yds. 3 qrs.

OF TIME.

L.yrs.	*mo.*	*w.*	*d.*	*h.*	*m.*	*sc.*	*Yrs.*	*d.*	*h.*
21	12	3	6	12	21	22	125	321	12
19	10	2	5	15	30	40	109	329	21
2	2	1	0	20	50	42	15	356	15

Practical Questions in Time.

1. A man hired a servant for 6 years, 4 months, 3 weeks, 6 days; and he stayed 7 years, 6 months, 3 weeks; how much longer did he stay than he first agreed to?

Ans. 1 yr. 1 mo. 3 w. 1d.

2. What is the difference between 21 years, 6 months, 3 weeks, 4 days, 12 hours; and 19 years, 9 months, 2 weeks, 6 days, 21 hours? Ans. 1 yr. 10 mo. 0 w. 4 d. 15 h.

CASE II.

(1) *To find the difference between two dates, when the time is expressed in years and calendar months and days.**

RULE.—Write down (2) the latest date first, (consider (3) whether the month is the first, second, or third, &c.; if it is January, write (4) 1 in the column of months; if it is February, write (5) 2, if March, (6) 3, &c.; and after writing the months, (7) write the day of the month) and then write down the other numbers (8) under those of the same denomination; (9) subtract, and the remainder is the difference between the two dates.

Questions.—1. What is required by Case 2d in Compound Subtraction?

* *Observe the following Table.*

January is the first month.	1st	July is the 7th month.	7th
February	2d	August	8th
March	3d	September	9th
April	4th	October	10th
May	5th	November	11th
June	6th	December	12th

—2. Which of the dates do you write down first?—3. What do you consider?—4. And if it is January what do you write?—5. If February, what?—6. If March, what?—7. After writing the months, what next?—8. Where do you write down the other numbers?—9. What is the last step in this case.

EXAMPLES.

1. What is the difference of time between the 16th day of February 1800; and the 19th day of May 1806?

Years.	*mo.*	*d.*
1806	5	19
1800	2	16
6	3	3 Ans.

NOTE.—In this example the latest year is written first, May being the 5th month, 5 is put in the column of months, and the day of the month in the column for days; and then the other denominations are put under those of the same name.

2. How long between the 15th day of June 1729, and the 30th day of August 1806, inclusive?

Ans. 77 years, 2 months, 15 days.

3. I was born the 7th day of May, 1777; what is my age, it being the 20th day of June 1810?

Ans. 33 years, 1 month, 13 days.

4. A. gave B. his note on interest, dated the 21st of August 1804; on the 29th day of November 1809, the note was paid; I demand the time that the note was at interest?

Ans. 5 years, 3 months, 8 days.

DECIMAL FRACTIONS.

As the present currency of the United States is calculated wholly by decimals, it is absolutely necessary they should be perfectly understood by the pupil.

DEFINITION.—Decimal Fractions are (1) parts of whole numbers, and are separated therefrom (2) by a point called the separatrix; thus 12·5; which is 12 and 5 tenths, or 12½; all the figures which stand at the left of the separatrix are (3) whole numbers; those on the right are (4) fractions. An unit is supposed to be divided into (5) ten equal parts, and

(6) the first figure on the right of the separatrix expresses the number of those parts; again, one of these parts is supposed to be divided into (7) ten more equal parts; and (8) the second figure in decimals expresses the number of those parts, &c. Thus decimals (9) decrease in a tenfold proportion, as they depart from the separatrix; (10) ·5 is 5 tenths; ·05 is 5 hundredths; and ·005 is thousandths, &c. Cyphers placed at the right hand of decimals (11) do not alter their value; ·5 ·50 and ·500 are decimals of the same value and equal to ·5, but cyphers placed at the left (12) diminish their value in a tenfold proportion.

Questions.—1. What are Decimal Fractions?—2. How are they separated from whole numbers?—3. What are those figures called which stand at the left of the separatrix?—4. What those on the right?—5. Into how many parts is the unit supposed to be divided?—6. What figure expresses those parts?—7. How many parts is one of these ten parts divided into?—8. What figure expresses those parts?—9. Do decimals increase or decrease as they depart from the separatrix, and in what proportion?—10 Give an example.—11. Do cyphers placed at the right hand of decimals alter their value?—12. Placed at the left, what effect do they have?

DECIMAL NUMERATION TABLE.

Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths	Millionths.
7	6	5	4	3	2	1	·1	2	3	4	5	6
	6	5	4	3	2	1	·1	2	3	4	5	
		5	4	3	2	1	·1	2	3	4		
			4	3	2	1	·1	2	3			
				3	2	1	·1	2				
					2	1	·1					
						1						

NOTE.—By the above table it is evident that the value of figures *increase* in whole numbers, as they depart from the

separatrix; and in the fractions, the value of the figures *decrease* as they depart from the separatrix. This should be illustrated by the Instructer and the pupil required to repeat the table.

ADDITION OF DECIMALS.

RULE.—Write down the several sums to be added in such a manner, (1) that the units in the whole numbers may stand under units, &c. and tenths in decimals may stand under tenths, &c. Add (2) as in Simple Addition, and (3) from the right hand of the sum total, point off as many figures as are equal to the greatest number of Decimal Figures in either of the sums added; those on the left will be whole numbers, and those on the right decimals.

Questions.—1. In what manner should the several sums in Addition of *Decimals* be set down?—2. How do you add?—3. What method do you take to ascertain which are whole numbers and which decimals?

EXAMPLES.

27·123	341·346	74·789	7·123
1·0341	91·2	7·61741	1·2
·1345	2·023	·214	10·32
28·2916	434·569	82·62041	18·643

Practical Questions.

1. Add 504·29, 64·1, 28·09, and 55·6 together.
Ans. 64·708.
2. Add $1327·64 cents; $2341·96 cents 9 mills; and $1572·21 cents together. Ans. $5241·819.
3. Add 46·969, 6·01 and ·946 together.
Ans. 53·925.

NOTE.—Dimes, cents, and mills are decimals of a dollar; one dime is one tenth, one cent is one hundredth, one mill is one thousandth: therefore the addition of American or Federal money, is the addition of decimals.

SUBTRACTION OF DECIMALS.

RULE.—(1) Write the largest number down first, and the smallest under it, in the same order as in Addition, then subtract the same as in Simple Subtraction.

Questions.—1. What is the general rule for subtraction of decimals?

EXAMPLES.

234·101	91·91	141·1	7·00041
145·164321	81·11234	98·3	5·14864
88·936679			

MULTIPLICATION OF DECIMALS.

RULE.—Multiply (1) exactly as in whole numbers, and (2) from the right of the product point off as many figures for decimals as are equal to the decimal figures in the multiplicand and multiplier counted together. If at any time there are not so many figures in the product as this rule requires, (3) supply the defect by prefixing cyphers to the left of the product.

Questions.—1. How do you multiply decimals?—2. How are the decimal places in the product ascertained?—3. When there are not so many figures in the product as this rule requires, what do you do?

EXAMPLES.

11·23	·1213
117·8	·1321
8984	1213
7861	2426
1123	3639
1123	1213
1322·894 pro.	·01602373 pro.

NOTE.—In the first example there being three decimal figures in the two factors, three figures are pointed off for decimals in the product; in the second example there are

eight decimal figures in both factors, and the product consists only of seven figures, therefore I prefixed one cypher and pointed all off for decimals.

No	*Multiplic.*			*Multiplier.*		*Product*
1.	1·23	×	by	17·8	=	21·894
2.	10·4	×	by	123	=	1279·2
3.	123	×	by	1·45	=	178·35
4.	·002	×	by	·661	=	·001322

Practical Questions.

1. How much is the product of 29·5 multiplied by ·96? Ans. 28·320.

2. How much will 1000 pounds of butter come to, at 11½ cts. per pound? Ans. $115.

3. How much will 64·5 bushels of corn come to, at $1.12 cts. per bushel? Ans. $72·24 cts.

DIVISION OF DECIMALS.

RULE.—Divide (1) exactly as in whole numbers, and (2) from the right hand of the quotient point off as many figures for decimals, as the decimal figures in the dividend exceed in number the decimal figures in the divisor; if there are not so many figures in the quotient as this rule requires, the defect must be supplied (3) by prefixing cyphers to the left of the quotient; if there are more decimal figures in the divisor than in the dividend, (4) place as many cyphers to the right hand of the dividend as will make them equal; and the quotient is (5) whole numbers till the dividend is all brought down; if a remainder still remains, (6) annex cyphers and continue the division; and the quotient thence arising will be decimals.

Questions.—1. How do you divide in decimals?—2. From which hand of the quotient do you point off for decimals, and how many?—3. If there are not so many figures in the quotient as this rule requires, how must the defect be supplied?—4. How do you divide when there are more decimal figures in the divisor than in the dividend?—5. What will the quotient then be?—6. What if there be still a remainder?

EXAMPLES.

1. It is required to divide 34·21 by 12·1.

12·1)84·21(2·8 *Ans.*
242

1001
968

33 rem.

NOTE.—There being one more decimal figure in the dividend than in the divisor, one figure is pointed off from the right of the quotient for decimals; cyphers might have been annexed to the remainder, and the quotient carried to a greater degree of exactness.

2. It is required to divide 1302 by ·101.
·101)1302·000(12891·089 Ans.

NOTE 2.—In this example the dividend is whole numbers, and the divisor has 3 figures in decimals. I therefore annexed 3 cyphers to the right of the dividend before I divided; and after the dividend was all brought down, I annexed 3 cyphers to the remainder, and continued the division, and got the decimal part of the quotient, viz. ·089.

3. Required to divide 263·146 by 1320.
1320)263·146(·199$\frac{4\ 6\ 0}{1\ 3\ 2\ 0}$ Ans.

NOTE 3.—In this example there are 3 decimal figures in the dividend, and the divisor is whole numbers; therefore I pointed off 3 figures from the quotient, for decimals.

4. Required to divide ·4567 by 333.
333)·4567(·0013$\frac{3\ 3\ 3}{3\ 3\ 3}$ Ans.

NOTE 4.—In this example, there are 4 figures in decimals in the dividend, and the divisor is whole numbers; the quotient has but 2 figures; I therefore prefixed 2 cyphers to the left of the quotient, and pointed all off for decimals.

5. Divide ·07912 by ·111. Ans. ·71.†*
6. Divide ·00444 by 222. Ans. ·00002.
7. Divide ·9023 by ·11. Ans. 8·202†
8. Divide 15·735 by 12. Ans. 1·311†.

* This mark (†) signifies that there is a remainder which is omitted in the answer.

REDUCTION OF DECIMALS.

CASE I.

(1) *To reduce Vulgar Fractions to Decimals.*

RULE.—(2) Annex cyphers to the numerator, and divide by the denominator; the quotient is the decimal sought.

NOTE.—A vulgar fraction is expressed thus $\frac{5}{6}$; the top figure is (3) a remainder left after division and is called the numerator; the bottom figure is (4) a divisor used in division, and is called the denominator.

Questions.—1. What does Case 1st in Reduction of Decimals require?—2. How do you reduce Vulgar Fractions to Decimals?—3. What is the top figure of a Vulgar Fraction, and what is it called?—4 What the bottom figure, and what called?

EXAMPLES

1. Reduce $\frac{5}{6}$ to a decimal of the same value

6)5·000

·833† Ans.

2. Reduce $\frac{1}{2}$ to a decimal of the same value. Ans. ·5
3. Reduce $\frac{1}{4}$ to a decimal of the same value. Ans. ·25.
4. Reduce $\frac{3}{4}$ to a decimal of the same value. Ans. ·75.

CASE II.

(1) *To reduce numbers having different denominations to decimals.*

RULE.—Write down the several denominations (2) under each other, (3) beginning with the lowest and ending with the highest; (4) on the left of these draw a perpindicular line, and on the left of the line, place such numbers for divisors, against each denomination, as it takes of the lowest to make one of the next higher: (5) annex cyphers to the top figure and divide by the number standing against it on the left of the line; set the quotient (decimally) (6) against the next denomination below; (7) divide the next below by the number standing against it, and set the quotient as before; proceed thus through all the denominations, the last quotient is the decimal sought.

Questions.—1. What does Case 2d in Reduction of Decimals require?—2. How do you write down the several denominations?—3. With which do you begin and end?—4. What is the next step?—5. How then do you proceed?—6. Where do you set the quotient?—7. What step do you then take?

EXAMPLES.

1. Reduce 12 pwt. 16 gr. to the decimal of a pound, Troy weight.

24 | 16·0000
20 | 12·6666†
12 | 0·6333†

·0527† Ans.

2. Reduce 6 oz. 12 pwt. 15 grs. to the decimal of a pound, Troy weight.

24 | 15·000
20 | 12·625† 10
12 | 6·6312

·5526 Ans.

3. Reduce 2 cwt. 3 qrs. 12 lb. 12 oz. 11 dr. to the decimal of a ton, Avoirdupois weight.

16 | 11·
16 | 12·
28 | 12·
4 | 3·
20 | 2·

·1432† Ans.

4. Reduce 3 qr. 3 na. to the decimal of a yard, Cloth measure.

4
4

·9375 Ans.

5. Reduce 2 miles 6 fur. 20 rods, 12 ft. 6 in. 2 bar. to the decimal of league, Long measure. Ans. ·9382†.

6. Reduce 3 roods, 30 rods, 29 yds. to the decimal of an acre, Land measure. Ans. ·9434†.

7. Reduce 56 gal. 3 qts. 1 pt. to the decimal of a hhd. allowing 63 gal. to a hhd. Ans. ·9027.

8. Reduce 39 ft. 1727 in. to the decimal af a ton of round timber. Ans. ·9999†.

9. Reduce 1 pk. 2 qts. 1 pt. to the decimal of a chaldron. Ans. ·0091†.

10. Reduce 12 mo. 3 wks. 6 dys. 23 hrs. 59 min. 12 sec. to the decimal of a year. Ans. ·9999†.

11. Reduce 2 min. to the decimal of a month, allowing 28 days to a month. Ans. ·00004†.

12. Reduce 1 pint to the decimal of a hhd. Wine measure. Ans. ·0019†.

13. Reduce 1 week to the decimal of a year, allowing 13 months to a year. Ans. ·01923†.

14. Reduce 1 minute to the decimal of a year, allowing 360 days to a year. Ans. ·000001.

15. Reduce 1 dram to the decimal of a ton, Avoirdupois weight. Ans. ·000001†.

NOTE.—A decimal table of weights and measures might be made by case II. which would be very convenient in a counting room, in which the several denominations in weights and measures might be reduced to the decimal of their respective integers in the following manner.

CLOTH MEASURE.

The parts of a yard reduced to the decimal of a yard.

1	qr.	is	·25	1	na.	is	·0625
2	"	"	·5	2	"	"	·125
3	"	"	·75	3	"	"	·1875

Use of such a Table.

Suppose it were required to rednce 3 qr. 1 na. to the decimal of a yard; Take the decimal of 3 qr. from the table, and the decimal of 1 na. and add them together, their sum will be the decimal sought.

The decimal of 3 qr. is ·75
The decimal of 1 na. is ·0625

·8125

CASE III.

(1) *To express the value of any decimal in the several denominations of the integer.*

RULE.—(1)Multiply the decimal by that number which

it takes of the next less denomination to make one of that denomination in which the decimal is given, and point off from the right hand of the product as many figures for decimals as is required by the rule in multiplication of decimals; then (3) multiply the last decimal by the next denomination, and so on to the last; the figures on the left of the separatrix are (4) the value of the decimal in the denominations of the integer.

Questions.—1. What does case 3d in Reduction of decimals require?—2. What is the 1st step in this case?—3. What the 2nd.?—4. What do those figures on the left of the separatrix express?

EXAMPLES.

1. Required the value of ·0527 of a lb. Troy weight.

lb. ·0527
12

oz. ·6324
20

pwt. 12·6480
24

grs. 15·5520

Ans. 12 pwt. 15† grs.

2. Required the value of ·5526 of a lb. Troy weight.
Ans. 6 oz. 12 pwt. 14† grs.

3. What is the value of ·1432 of a ton?
Ans. 2 cwt. 3 qr. 12 lb. 12 oz. 4† dr.

4. What is the value of ·9017 of a lb. Apothecaries' weight? Ans. 10 oz. 6 dr. 1 sc. 13† grs.

5. What is the value of ·9376 of a yard, Cloth measure?
Ans. 3 qr. 3† na.

6. What is the value of ·9436 of a league, Long measure? Ans. 2 mi. 6 fur. 25 rds. 4 yds. 2 ft. 1† in.

7. What is the value of ·9436 of an acre, Land measure?
Ans. 150† rods.

8. What is the value of ·9999 of a ton of round timber?
Ans. 39 ft. ·1721† in.

9. What is the value of ·0099 of a chaldron?
Ans. 1 pk. 3 qts.

10. What is the value of ·0002 of a month, allowing 28 days to a month? Ans. 8 min, 3† sec.

REDUCTION.

DEFINITION.—Reduction teaches (1) to reduce one denomination to another, and retain the same value. It is of (2) two kinds, Descending and Ascending.

REDUCTION DESCENDING.

Reduction is called descending (3) when it is required to reduce a larger to a smaller denomination, and it is then performed by multiplication.

RULE.—(4) Multiply the highest denomination given, by such a number as will reduce it to the next lower, observing to bring in the parts of the same name, thus continue till you have reduced it to the denomination required.

Questions.—1. What does reduction teach?—2. How many kinds of reduction—and what are they?—3. When is reduction called descending, and how is it performed?—4. What is the rule for its operation?

EXAMPLES.

1. It is required to reduce 444 dols. 16 cts. 5 m. into mills.

dols.	*cts.*	*m.*
444	16	5
100		
44416 cts.		
10		

Ans. 444165 mills.

2. In 57 dollars 93 cents 5 mills, how many mills?
Ans. 57935 mills.

3. In 4 cwt. 2 qrs. 20 lb. how many pounds, Avoirdupois weight. Ans. 524 lb.

4. In 20 acres, 29 rods, how many square rods?
Ans. 3229 rods.

5. How many inches from Exeter to Dover, it being 18 miles? Ans. 1140480 in.

6. How many inches will reach round the world, it being 360 degrees, each degree containing 69½ miles?
Ans. 1585267200 in.

7. How many solid feet in 20 cords of wood?
Ans. 2560 sd. ft.

8. How many pints are in 1 tun of wine?
Ans. 2016 pts.

9. How many hours in 33 years, 2 months, 12 days, allowing 13 months to a year? Ans. 289920 h.

10. In 29 yards, 2 quarters, 3 nails, how many na.?
Ans. 475 na.

11. In £214 18s. 6½d. how many farthings?
Ans. 206330.

12. In £64 19s. 6d. how many pence?
Ans. 15594.

REDUCTION ASCENDING.

DEFINITION.—Reduction is called ascending (1) when it is required to reduce a smaller, to a larger denomination; and it is then performed by (2) division.

RULE.—(3) Divide the given denomination by such a number as will make one of the next higher; and the last quotient by such a number as will make one of the next, &c. until you have reduced it to the denomination required.

Questions.—1. When is Reduction called Ascending?—2. By what principal rule is it performed?—3. What is the general rule for its operation?

EXAMPLES.

1. Reduce 444165 mills into dollars.

10)444165

10)44416 cts. 5 m.

10)4441 dimes, 6 cts.

Ans. 444 dolls. 16 cts. 5 m.

2. In 57935 mills, how many dollars, &c.
Ans. $57 93 cts. 5 m.

3. In 524 lb. Avoirdupois, how many cwt. &c?
Ans. 4 cwt. 2 qrs. 20 lb.

4. In 3229 rods, how many acres, Square measure?
Ans. 20 acres, 29 rods.

5. In 1140480 inches, how many miles Long measure?
Ans. 18 miles.

6. In 1585267200 inches, how many degrees, each containing 69 ½ miles? Ans. 360 degrees.

7. How many cords of wood are in 2560 solid feet?
Ans. 20 cords.

8. In 2016 pints of wine, how many tuns? Ans. 1 tun.

9. In 289920 hours, how many years, allowing 13 months to a year? Ans. 33 yrs. 2 mo. 12 d.

10. In 475 nails, how many yards?
Ans. 29 yds. 2 qr. 3 na.

11. In 206330 farthings how many pounds, &c.?
Ans. £214 18s. 6½d.

12. In 15594 pence, how many pounds, &c.?
Ans. £64 19s. 6d.

COMPOUND MULTIPLICATION.

DEFINITION.—Compound Multiplication is a rule much used in business; and teaches, (1) by having the price of one artiele given, to find the price of any number of the like kind.

Questions.—1. What does Compound Multiplication teach?

CASE I.

(1) *When the quaitity is whole numbers.*

RULE.—(2) Multiply the price and quantity together, the product is the answer; if there are dimes, cents and mills in the price, (3) point off for decimals according to the rule of multiplication of decimals.

Questions.—1. When is Case 1st in Compound Muntiplication applied? —2. What is the rule?—3. If there are dimes, cents and mills in the price, what do you do?

EXAMPLES.

1. How much will 1000 lbs. of butter come to at 11 cts. 5 mills per pound? 1000 lbs. × ·115=$115·000 Ans

NOTE.—In this example 11 cents and 5 mills are exactly equal to 115 thousandths of a dollar; therefore I multiplied the price by the quantity, the product was 115000; and according to the rule in the multiplication of decimals, I pointed off three figures from the right for decimals, the rest were dollars.

2. How much will 349 pounds of pork come to at 07 cents per pound? Ans. 24 dolls. 43 cents.

3. How much will an ox come to, whose weight and prices are as follows, viz.—The quarters 205·5, 202, 207 and 200 lbs. at ·06 cents; the hide 70 lbs. at ·07 cents; the tallow 66 lbs. at ·09 cents lb. Ans. 59 dolls. 71 cts. 0 m.

4. A drover sold 100 lambs in 5 separate lots, 20 to each; the first lot was sold for $1·96 cents each; the second for $1·90 cents; the third for $2·05 cents; the fourth for $2·10 cents; and the fifth at $2·25 cents; I demand the price of the whole. Ans. $203·20 cts.

BILLS OF PARCELS.

Exeter, April 10, 1811.

Mr. A. B. bought of Mr. C. D.

			Cts.	*Dolls. Cts.*
6	gallons of West-India rum	at	·96	5·76
7	do. of New-England do.	"	·55	3·85
12	lbs. of hyson tea	"	$1·25	15·00
19	pairs of shoes	"	·96	18·24
20	lbs. of coffee	"	·28	5·60
29	lbs. of rice	"	·04	1·16
90	lbs. of cotton	"	·23	20·70
15	doz. of biscuit	"	·15	2·25
				$72·56

Received payment in full.

Signed C. D.

Exeter, April 10, 1811.

Mr. J. K. of Portsmouth, bought of Mr. G. H.

			Cts.	*Dolls. Cts.*
1000	gallons of West-India rum	at	$1·25	
960	do. of molasses	"	·60	
90	cwt. of brown sugar	"	9·50	
15	barrels of flour	"	8·25	
3	crates of ware	"	40·30	
2	cwt. of iron	"	5·60	
69	gallons of Holland gin	"	1·40	
20	cases of knives and forks	"	·75	
650	lbs. of coffee	"	29	
				$3200·95

Received payment by note, payable in 90 *days.*

Signed G. H.

CASE II.

(1) *When there are fractions in the quantity, such as* $\frac{1}{8}$ $\frac{1}{14}$ $\frac{5}{6}$, *&c.*

RULE.—(2) Reduce the vulgar fraction to a decimal; then (3) multiply the quantity and prices together, point off for decimals according to rule; all that are on the left of the separatrix are (4) dollars; and all on the right are parts, viz. (5) dimes, cents, and mills.

Questions.—1. When is Case 2d in Compound Multiplication applied? —2. What is the 1st step in the rule?—3. What is the 2d and last st p?—4. What are those on the left of the separatrix called?—5. What those on the right?

EXAMPLES.

1. What will 102¾ yards of India cotton, come to at 29 cents per yard?

Yds. Cts. Dolls.cts.m.

¾=·75 then 102·75×·29= 29·79·7·5 Ans.

NOTE.—In this example ¾ reduced to a decimal, gave two decimal places in the quantity, and there were two decimal places in the price; of course there were four places of decimals in the product; the three first figures in decimals, are cents and mills, and the other is parts of a mill, which in business is not worth noticing.

2. What will 21½ yards of velvet come to, if 1 dollar 25 cents are paid for 1 yard? Ans. $26·87 cts. 5m.

3. What will 6$\frac{7}{12}$ of a yard of broadcloth come to, at 6 dollars 75 cents per yard? Ans. $45·56 cts. 2½ m.

CASE III.

(1) *When there are several denominations in the quantity.*

RULE.—(2) Reduce the several denominations to the decimal of the highest, by case 2, in reduction of decimals; * then (3) multiply the price and quantity together, point for decimals according to rule; the answer will be (4) dollars cents and mills, and parts of mills.

Questions.—1. When is Case 3d in Compound Multiplication applied?—2. What is the 1st step in this rule?—3. What the 2nd and last step?—4. What will the answer be in?

* To this Case it will be necessary that the scholar be refered, and required to give the steps of it explicitly.

EXAMPLES.

1. How much will 3 cwt. 2 qr. 20 lb. of sugar come to, at $12·10 cts. per cwt.?

28 | 20.000†8

4 | 2·714

cwt. 3·678×12·10=$44·50 cts. 3 m. Ans.

NOTE.—2 qr. 20 lb. reduced to the decimal of a cwt. is equal to 678 thousandths, which I annexed to the hundreds, and multiplied by the price.

2. How much will 173 cwt. 3 qr. 27 lb. of iron come to, at $5·51 cts. per cwt.? Ans. $958.69† cts.

3. How much will 356 acres, 1 rood and 7 rods of land come to, at $17·21 cts. per acre? Ans. $6131·80 cts. 2†m.

4. What will 64 tons, 3 cwt. of potash come to, at $84·50 cts. per ton? Ans. $5420·67 cts. 5 m.

5. What will 38 cwt. 3 qr. 21 lb. of sugar come to, at $11·52 cts. per cwt.? Ans. 448·55 cts. 4†m.

6. What will the wages of a servant come to for 70 yrs. 3 mo. 15 d. allowing 26 days to a month, and 12 months to a year, at $6·71 cts. 5 m. per year?

Ans. $472·05 cts. 1 m.

CASE IV.

(1) *Rules to find the amount of boards, plank, and other articles that are sold by the thousand, or hundred.*

RULE.—If the article is sold by the thousand, place a separatrix (2) between the thousands and hundreds; the hundreds, &c. are decimals of a thousand; if the article is sold by the hundred, place the separatrix (3) between the hundreds and tens; the tens are decimals of an hundred; then (4) multiply the price and quantity together, the product pointed according to the rule of multiplication of decimals, will be the answer required.

Questions.—1. To what is Case 4th in Compound Multiplication applied?—2. If the article is sold by the thousand, where do you place the separatrix?—3. If sold by the hundred, where placed?—4. What is the last step in this Case?

EXAMPLES.

1. How much will 10751 feet of boards come to, at $11·21 cents per thousand?

Feet. *D. cts.* *D. cts. m.*
10·751×11·21=120·51·8† Ans.

2. How much will 1100 feet of plank come to, at 33 dollars per thousand? Ans. $36·30 cts.

3. How much will 941 feet of clear boards come to, at $25 per thousand? Ans. $23·52 cts. 5 m.

4. How much will 627 hoops come to, at $1·27 cts. per hundred 6·27×1·27=$7·96† Ans.

CASE V.

(1) *To multiply by fractional parts.*

RULE.—(2) Multiply by the numerator and divide by the denominator, the quotient is the answer.

Questions.—1. What does Case 5th in Compound Multiplication teach? —2. Give the rule.

EXAMPLES.

1. What is the value of $\frac{8}{10}$ of a ship, worth $10000?
10000×8=80000÷10=$8000 Ans.

2. What is $\frac{4}{7}$ of a house worth, which is valued at $3470?
Ans. $2478·57·1†.

CASE VI.

(1) *Lawful money, Weights and Measures.*

RULE.—If the number is less than 12, (4) multiply the weight or price of one by the number; observing (3) to carry as in Compound Addition; if the number is more than 12, (4) multiply by two such numbers as will make the number required; if there are no two such numbers, (5) multiply by two such numbers as will come nearest, and for the other numbers, add or subtract as the case may be.

Questions.—1. Of what does Case 6th in Compound Multiplication treat? —2. If the number or quantity is less than 12, what do you do?—3. What do you observe?—4. If the number or quantity is more than 12, what do you do?—5. If there are no two such numbers in the Table as will make the number given, what do you do?

EXAMPLES.

1. What is the weight of 9 casks of raisins, each weighing 3 cwt. 2 qr. 12 lb. 6 oz.

Cwt.	*qr.*	*lb.*	*oz.*	
3	2	11	6	
			9	
32	1	18	6	Ans.

2. What is the weight of 56 casks of raisins, each weighing 1 cwt. 2 qrs. 12 lb.? Ans. 90 cwt.

3. What is the weight of 105 casks of tobacco, each weighing 3 cwt. 1 qr. 7 lb.? Ans. 347 cwt. 3 qr. 7 lb.

4. How much will 12 oxen come to, at £10 16s. per ox? Ans. £129 12s.

5. How much will 272 barrels of flour come to, at £2 10s. 6d. per barrel? Ans. £686 16s.

RULE 2.—(1) Reduce the price or weight to the lowest denomination mentioned, then (2) multiply by the quantity as in Simple Multiplication, and then (3) reduce the product into its proper terms again.

Questions.—1. What is the 1st. step in rule 2nd, in Case 6th in Compound Multiplication?—2. What is the 2nd step?—3. What is the 3d step?

EXAMPLES.

1. How much will 27 thousand of boards come to, at £2 10s. 6½d. per thousand?

£.	s.	d.	
2	10	6½	
20			
50			
12			
606			
4			
2426			
27			
16982			
4852			
4)65502(			
12)16375(7			
20)1364(4			

£68 4s. 7½d. Ans.

2 How much will 51 cwt. of iron come to, at £2 1s. 6d. per hundred? Ans. £105 16s. 6d.

COMPOUND DIVISION.

DEFINITION.—Compound Division (1) teaches, by having the price of several things given, to find the price of one.

Questions.—1. What does Compound Division teach?

CASE I.

FEDERAL MONEY.

(1) *When the quantity is an even number.*

RULE.—Use the price for (2) a dividend, and the quantity for (3) a divisor; (4) divide, obseving the same rule as in the division of decimals, and the quotient is (5) the answer, or price of one.

Questions.—1. When is Case 1st in Compound Division used?—2. For what is the price used?—3. For what the quantity?—4. How then do you proceed, and what do you observe?—5. What does the quotient express?

EXAMPLES.

1. If 24 bushels of rye cost $21, how much is 1 bushel of it worth?

Dol. *Cts.*
24)21·000(·875=87 cts. 5 m. Ans.

2. If 340 bushels of rye cost $456·25 cts.; I demand the price of 1 bushel? Ans. $1·341$\frac{31}{34}$.

3. If 2020 lb. of cheese cost $181·80 cts. what did it cost per pound? Ans. $0·09 cts.

CASE II.

(1) *To find the price per cwt. when there are parts in the quantity, such as cwt. qr. &c.*

RULE.—(2) Reduce the smaller denominations to the decimal of a cwt. (by case 2d in reduction of decimals) then (3) divide the price by the quantity, as in Case 1st; (4) the quotient, pointed according to the rule of decimals, will be the answer.

Questions.—1, What is the use of Case 2d in Compound Division?—2. What is the first step in the rule?—3. What is the 2d step?—4. What is the 3d, and last?

EXAMPLES.

1. If 2 cwt. 3 qr. of sugar cost $45·26 cts.; I demand the price of 1 cwt.

Cwt. qr. cwt. D. cts.
2 3=2·75)45·26($16·458$\frac{5}{7}\frac{0}{7}\frac{0}{5}$ Ans.

NOTE.—In this question, 3 qr. reduced to the decimal of a cwt. is equal to ·75, which I annexed to the 2 cwt. for a divisor; and divided $45·26 cts. by it, and got 16 dollars in the qnotient; I then annexed three cyphers to the remainder, and continued the division, and thereby got the remaining part of the quotient, viz. ·458 $\frac{5}{7}\frac{0}{7}\frac{0}{5}$.

2. If 15 cwt. 2 qr. of rice were sold for $54·25 cts.; I demand the price of 1 cwt. Ans. $3·5 dimes.

3. If 3 cwt. 2 qr. 14 lb of iron cost $16·67·5; I demand the price per cwt. Ans. $4·6 dimes.

CASE III.

(1) *When cwt. qrs. and lb. are given, to find the price per lb.*

RULE.—(2) Find the price per cwt. by case 2d; then (3) divide the price per cwt. by 112, the quotient will be the price per lb.: Or, (4) reduce the quantity to pounds for a divisor, using the price for a dividend.

Questions.—1. When is Case 3d in Compound Division used?—2. What is the 1st step in this rule?—3. What is the 2d step?—4. What other method can you have for finding the price per pound?

EXAMPLES.

1. If 3 cwt. 2 qr. 14 lb. of iron cost $16·67 cts. 5 m., I demand the price per pound.

Cwt. qr. lb. cwt. D.cts.m. D.d. c.m.
3 2 14=3·615)16·675(4·6÷112=·041$\frac{9}{11}$ Ans.
14500

21750
21750

CASE IV.

LAWFUL MONEY.

RULE.—(1) Divide the price by the quantity, the quotient is the answer; if pounds, shillings, and pence are in

the price, (2) divide the highest denomination by the quantity, observing to reduce the remainder to the next denomination, adding in the parts of the same name, and continue the division through all the denominations to the lowest.

Questions.—1. What is the 1st step in the rule in Case 4th in Compound Division?—2. If pounds shillings and pence are in the price, how do you divide?

EXAMPLES.

1. If 24 oxen cost £.246 16*s.* 6*d.*, how much must be paid for one?

£. s. d.
24)246 16 6 (£.10 5*s.* 8½*d.*
24
6
20
24)136(5*s.*
120
16
12
24)198(8
192
6
4
24)24(1
24

CASE V.

(1) *To divide by fractional parts.*

RULE.—To divide by fractional parts, is the same as multiplying by them; (2) see case 5th, Compound Multiplication.*

Questions.—1. What does Case 5th in Compound Division teach?—2. What is the rule?

*The scholar should here be required to turn to this Case and give the rule required.

EXAMPLES.

1. What is the value of ¼ of an house, which is worth $3000? 3000×3=9000÷4=$2250. Ans.

CASE VI.

(1) *When the number of shares are unequal.*

RULE.—(2) Divide the sum by the number of simple shares, the quotient will be the share of the first, which (3) multiply by so many as the second has more than the first, and thus continue till you have found all the shares.

PROOF.—(4) Add all the shares together, and if the sum is equal to the sum divided, the work is right.

Questions.—1. When is Case 6th in Compound Division used?—2. What is the first 1st step in the rule?—3. How then do you proceed?—4. What is the method of proof?

EXAMPLES.

1. Divide $373·50 cts. among A. B. and C., in such a manner, that B. may have twice as much as A. and C. twice as much as B.

A. has 1 simple share.
B. " 2 " shares.
C. " 4 " shares.

7 number of simple shares.

$373·50 ÷7=$53·35$\frac{5}{7}$, A's.
A's. 53·35$\frac{5}{7}$ ×2=106·70$\frac{10}{7}$, B's.
B's. 106·70$\frac{10}{7}$×2=213·40$\frac{20}{7}$, C's.

Proof.—$373·50 cts.

2. Divide $1089·33 cents among 4 persons, and give the second, three times as much as the first; the third, four times as much as the second, and the fourth, five times as much as the third.

Ans. A's. $14·33†; B's. $42·99†; C's. $171·96†; and D's. $859·80 cts.; twenty five cents being lost in fractions.

CASE VII.

(1) *When the shares are not equal, but increases by a certain ratio, as 1, 2, 3, 4, 5, &c.*

RULE.—(2) Divide the sum by the number of persons, the quotient is a mean, or middle share; (3) from the middle share subtract the ratio, the remainder is the next share that is less; (4) from the last found share subtract the ratio until you have found all the shares that are less; (5) to the middle share add the ratio, the sum is the next share that is larger; (6) to the last found share add the ratio till you have found all the shares that are larger. If the number of persons is an even number, as 4, 6, 8, &c. (7) divide as above, and from the quotient subtract half the ratio and the remainder is one share—and add half the ratio to the quotient, the sum is another share; these two shares are the two middle shares; (8) for the shares that are less continue to subtract the ratio, and for those that are larger, add the ratio, till you have found all the shares.

PROOF.—(9) Add together the several shares found, and if the sum total is equal to the sum divided, the work is right.

Questions.—1. When is Case 7th in Compound Division used?—2. How do you find the mean or middle share?—3. How the next less?—4. How the other smaller shares?—5 How do you find the next share larger than the middle?—6. How the other larger shares?—7. If the number of persons are even, how do you find the two mean or middle shares?—8. How do you find those shares that are less, and those that are larger than the two middle shares?—9. What is the method of proof?

EXAMPLES.

1. Divide $600 among 5 persons, in such a manner that B. may have two more than A., and C. two more than B., &c.

$$\$600 \div 5 = \left\{ \begin{array}{ll} 116 & \text{A's share,} \\ 118 & \text{B's } \text{“} \\ 120 & \text{C's } \text{“} \\ 122 & \text{D's } \text{“} \\ 124 & \text{E's } \text{“} \end{array} \right\} \text{Ans}$$

Proof.—$600.

2. Divide $640 among 7 persons, in such a manner that the second shall have one more than the first, the third one more than the second, &c.

Ans. A's share, $88·42$\frac{6}{7}$; B's $89·42$\frac{6}{7}$; C's $90·42$\frac{6}{7}$; D's $91·42$\frac{6}{7}$; E's $92·42$\frac{6}{7}$; F's $93·42$\frac{6}{7}$; G's $94·42$\frac{6}{7}$.

3. Divide $1600 among four persons, in such a manner that the second may have one more than the first, the third one more than the second, &c.

1600·00÷4=400·00 the middle share.

dol.	*dol. cts.*
399·50—100 cts. ratio	=398·50 A's share.
400·00— ·50 cts. half ratio	=399·50 B's share.
400·00+ .50 cts. half ratio	=400·50 C's share.
400·50+ 100 cts. ratio	=401·50 D's share.
	$1600·00 Proof.

AVERAGE JUDGMENT.

DEFINITION.—Average Judgment, is (1) the mean, or middle judgment, of several persons, who are appointed to appraise any particular property.

RULE.—(2) Add together the several sums which the commodity is appraised at, for a dividend; and use the number of appraisers for a divisor; divide, and the quotient will be the mean, or middle judgment required.

Questions.—1. What is Average Judgment?—2. What is the general rule?

EXAMPLES.

1. What is the value of a piece of land, which is valued by A. at $10; by B. at $11·50; by C. at $12·30; and by D. at $13·40 cts. per acre?

A. 1.	-	$10·00
B. 1.	-	11·50
C. 1.	-	12·30
D. 1.	-	13·40
Appraisers 4		4)47·20

Ans. $11·80 cts.

2. A. B. C. D. E. and F. were appointed to appraise a certain estate; they appraised it as follows, viz. A. at $3470; B. at $3650; C. at $3700; D. at $3500; E. at $3400; and F. at $3600; I demand the value of the estate.

Ans. $3553·33 cts. 3⅓ m.

8. M. N. O. and P. appraised the ship Lucy as follows, viz. M. at $6700; N. at $9000; O. at $8750; and P. at $7380; what is the middle judgment? Ans. $7957·5 d.

COMPOUND AVERAGE JUDGMENT.

Compound Average Judgment is (1) *when the judgment of the referees is partly on one side of the equality and partly on the other.*

RULE.—(2) Subtract one side from the other, and divide the remainder by the number of referees, and the quotient will be the average judgment.

Questions.—1. When is average judgment said to be compound?—2. Give the rule for its operation?

EXAMPLES.

1. A. and B. wishing to exchange horses and disagreeing as to the conditions, referred the matter to X. Y. and Z. by whose judgments they agreed to abide, which were as follows, viz.—X. said A. should pay B. $8, and Y. said A. should pay B. $6; but Z. said B. should pay A. $5. What is the average judgment? Ans. $3 B. receives.

2. E. and F. proposed to swap watches—agreed to refer it to A. B. C. and D. to say how they should exchange A. marked that E. should have $4; B. said E. should have $5; C. said E. should have $2; but D. said F. should have $3,50; which receives the boot, and how much?

Ans. E. $1·87·5.

SINGLE RULE OF THREE DIRECT.

DEFINITION.—The Single Rule of Three Direct teaches, (1) by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second has to first. (2) When more requires more, or less requires less, the proportion is direct. More requiring more, is (3) when the third term is greater than the first, and the sense of the question requires that the fourth term should be greater than the second; less requiring less, is (4) when the third

term is less than the first, and the sense of the question requires that the fourth term should be less than the second.

RULE.—(5) State the question, or arrange the three given numbers in such order, that the one which asks the question may stand in the third place;* that number which is of the same name with the third, must possess the first place; the remaining number (which is always of the same name with the number required) must possess the middle place. (6) Reduce the first and third terms, or numbers, into the same denomination; and reduce the middle number, or term, into the lowest denomination mentioned; then (7) multiply the second and third terms together, and divide the product by the first; the quotient will be (8) the answer, or fourth term sought; and always will be (9) of the same denomination as the middle term was in when it was multiplied with the third term; and may be reduced to any other denomination required.

Questions.—1. What does the Single Rule of Three Direct teach?—2. When is proportion direct?—3. When does more require more?—4. When does less require less?—5. What is the first step in the Rule?—6. What the second step?—7. What the third?—8. What will the quotient be?—9. Of what denomiation will it be, and to what may it be reduced?

RULE OF THREE IN DECIMALS.

RULE.—State the question (1) as in the Rule of Three Direct; prepare the terms (2) by reducing the smaller denominations to the decimal of the highest; observing (5) that the Integer in the first and third terms are in the same denomination: (4) multiply and divide as in the Rule of Three Direct, and point off for decimals as is required in the rule of multiplication and division of decimals.

Questions.—1. How do you state the question in the Rule of Three in Decimals?—2. How do you prepare the terms?—3. What do you observe? —4. How then do you proceed?

NOTE.—As the currency of the United States is a decimal calculation, it becomes most necessary to calculate in that way; but I have done the questions in the Rule of Three by both methods, therefore one will prove the other.

*The third term always asks a question, and is generally preceded by some such words as, What will? How much? How far? How long? How soon? What is? Where will? &c.

EXAMPLES.

1. If ten sheep are worth $22·22: what are 19 of the kind worth?

	Sheep.	*D. cts.*	*Sheep.*
As	10 :	22·22 : :	19

19

19998
2222

10)422·18

42·218=$42·21 cts. 8 m. Ans.

NOTE.—In the first question 19 sheep is the number that asks the question, and is placed in the third place; 10 being of the same name, viz. sheep, it is placed in the first place, and the remaining number, viz. $22·22 cents, is the number left, and is used for the middle term; and is of the same name with the number sought, viz. money.

The same question by direct proportion.

	Sheep.	*D. cts.*	*Sheep.*
As	10 :	22·22 : :	19

100

2222 cents.
19

19998
2222

10)42218

100)4221 $\frac{8}{10}$ cts.

$42·21 $\frac{8}{10}$ Ans.

NOTE.—In this example the first and third terms are of the same name—they need no reducing; there being cents mentioned in the middle term, I reduced the whole to cents; which I divided by 100 to bring them into dollars.

2. If 20 acres, 1 rood, 20 rods of land, cost $183·37 cts. 5 m.; what will 37 acres of the same kind be worth?

20 acres, 1 rood, 20 rods=20·375 acres.

Acres. dolls. acres.
As 20·375 : 183·375 : : 37 $333 Ans.

NOTE.—In this example I reduced the one rood 20 rods, to the decimal of an acre; then I multiplied and divided according to the rule of decimals.

The same question done by direct proportion.
Acr. rood. rds. D. cts. m. acres.
As 20 1 20 : 183·37·5 : : 37 Ans. $333.

NOTE.—In this example there being rods in the first term, I brought all the first term into rods, and also brought the third term into rods; and the middle term into mills; then multiplied and divided according to rule, the answer came in mills.

3. If 28 cwt. 3 qr. of sugar is worth $261·25 cts., I demand what must be given for 2 cwt. 1 qr. of the kind.
Ans. 24 dolls. 75 cts.

The same question done by direct proportion.
Ans. $24 75.

4. What will 4 hhd. of rum come to, containing as follows, viz. the first 101½, the second 96¾, the third 89½, and the fourth 111¾, gallons, if 6½ gallons cost $6·89?
Ans. $423·20 cts. 5 m.

The same question done by direct proportion.
Ans. $423·20 cts. 5 m.

5. If 1000 feet of boards are worth $11·12 cents, I demand the price of 17221 feet. Ans. $191·49 cts. 7†m.

The same question done by direct proportion.
Ans. $191·49 cts. 7†m.

6. If ½ of 6 be three, I demand the value of ¼ of 20.
Ans. 7·5.

The same question done by direct proportion.
Ans. 7½.

7. If 6½ dozen pigeons are worth ·65 cents, I demand the price of 29¾ dozen. Ans. $2·97 cts. 5 m.

The same question done by direct proportion.
Ans. $2·97·5.

8. If 65 bushels and 1 peck of corn were spent in a family of 6 persons, I demand the quantity that would be sufficient to support a family of 22 persons for the same time.
Ans. 239·25 bushels.

The same question done by direct proportion.
Ans. 239¼ bushels.

9. How long will 34261 lbs. of beef last an army of 600 men, allowing them to draw 4½ ounces each, and that 3 times per day? Days $67\frac{3}{8}\frac{4}{1}\frac{1}{6}\frac{5}{0}$ Ans.

The same question done by direct proportion.

Ans. $67\frac{3}{8}\frac{4}{1}\frac{1}{6}\frac{5}{0}$ days.

10. How many times would a wheel, that is 16 feet 3 inches in circumference, turn round in going round the world on the equator, allowing a degree there to contain 69½ miles? Ans. 8129575 $\frac{6}{1}\frac{3}{6}\frac{5}{2}\frac{5}{5}$.

The same question done by direct proportion.

Ans. 8129575 $\frac{7}{1}\frac{5}{6}\frac{5}{2}$

11. How far are the inhabitants of the equator carried in a minute, allowing the earth to make one revolution in 24 hours, and allowing a degree to contain 69½ miles?

Ans. 17 mi. 3 fur.

The same question done by direct proportion.

Ans. 17 mi. 3† fur.

12. A merchant failing in trade, is owing $6420·20 cents; his effects are worth $3142·75 cents; what will A. lose, to whom he owed $960·73 cents? Ans. $490·45.

The same question done by direct proportion.

Ans. $490·45 nearly.

13. A merchant failing, is owing $7500; his effects amount to $5640; what can he pay per cent?

Ans. $75·2† dimes

The same question done by direct proportion.

Ans. $75·20† cts

14. If the earth makes one revolution in one solar day, I demand the time that it is in passing one degree.

Ans. 4 minutes.

The same question done by direct proportion.

Ans. 4 minutes.

15. If the sun is 4 minutes in passing one degree, I demand the difference of time of its coming to the meridian, at two places which lie 20 degrees apart.

Ans. 1 hour 20 minutes.

The same question done by direct proportion.

Ans. 1 hour 20 minutes.

16. What is the insurance upon $472 dolls. at 3½ per cent?

Ans. $121·52 cts.

The same question done by direct proportion.

Ans. 121·52 cts.

17. If half an acre of land is worth $59·20 cents, I demand the price of 1¼ acres. Ans. $148.

The same question done by direct proportion.

Ans. $148.

18. I demand the value of $642 against an estate which can pay only ·69 cts. on the dollar. Ans. $442·98 cts.

The same question done by direct proportion.

Ans. $442·98.

19. There is a cistern having 4 cocks; the 1st empties it in 15 minutes; the 2d in 30 minutes; the 3d in 45 minutes; and the 4th in 60 minutes; In what time would it be emptied if they were all running together?

A's 15 : 1 : : 120 : 8
30 : 1 : : 120 : 4
45 : 1 : : 120 : $2\frac{2}{3}$
60 : 1 : : 120 : 2
$16\frac{2}{3}$

Cisterns min. cisterns min. sec.

As $16\frac{2}{3}$: 120 : : 1 : 7 12 Ans.

20. If £21 12s. be paid for building 60 rods of stone wall, how much must be paid for the building of 96 rods?

Ans. £34 11 s.

21. What must be given for $19\frac{1}{2}$ yards of broadcloth, if 3 yards of the like kind cost £19 11s. 6d.

Ans. £127 4s. 9d.

22. My horse and saddle are worth $40, but the horse is worth four times as much as the saddle; what is the value of each? Ans. Horse $32; saddle $8.

23. B. and C. depart for the same place and travel the same road; but B. goes 8 days before C. at the rate of 30 miles per day; C. follows at the rate of 40 miles per day; in what time and distance will he overtake B.?

Ans. 24 days; 960 miles.

24. If in 4 months I spend 3 months gain, how much can I lay up at the year's end, if I gain $150 every 6 months?

Ans. $75.

25. Bought 126 gallons of rum for 110 dollars, how much water must be added, to reduce the first cost to 75 cents per gallon? Ans. $20\frac{2}{3}$ gallons.

26. If 60 gallons of water in one hour, fall into a cistern containing 360 gallons, and by a pipe in the cistern 35 gallons run out in 40 minutes, in what time will it be filled?

Ans. 40 hours

METHOD OF MAKING TAXES.

The first thing requisite in making taxes is, (1) to know the rate at which polls and other rateable estate are valued by the statutes of the state.

NOTE.—As these rates are established by the Legislature of the state, they are often altered.

The value of Rateable Estates, as valued by the laws of New-Hampshire in 1830.

POLLS.—Each poll from 18 to 70 years of age, except those excused by the statute	-	at -	$1·10
HORSES.—Stallions wintered 3 winters, each		" -	10·00
Other horse kind wintered 5 winters		" -	·50
do. do. 4 do.		" -	.35
do. do. 3 do.		" -	·20
do. do. 2 do.		" -	·10
OXEN.—Oxen five years old, each ox		" -	·30
do. four years old, do.		" -	·20
COWS.—Cows wintered 5 winters each		" -	·15
OTHER NEAT STOCK.—All neat stock wintered 3 winters.		" -	·08
All neat stock wintered 2 winters		" -	·05
ORCHARD LAND.—So much orchard land as will produce 10 barrels of cider or perry is called an acre; and each acre is valued		" -	cts. ·20
TILLAGE LAND.—So much land as will produce 25 bushels of corn, or other grain equivlent, is accounted an acre; and eaeh acre is valued		" -	·20
MOWING LAND.—As much as will produce one ton of English hay yearly, or other hay equivalent, is accounted an acre, and each acre is valued		" -	·20
PASTURE LAND.—As much pasture land as will keep 1 cow (one year with another,) is called 4 acres; and each acre is valued		" -	·05

MILLS, &c.—These are estimated at one twelfth of their neat yearly income after deducting repairs, &c.

BUILDINGS, &c.—Of inhabitants and non residents, are valued at half of one per cent. of their real value.

STOCK IN TRADE.—Stock in trade at half of one per cent.

BANK SHARES.—All bank shares at ½ of one per cent.

CARRIAGES.—All carriages of pleasure are valued at half of one per cent. of their real value.

MONEY.—Money at interest, or on hand more than the owner pays interest for, is valued at ¾ of one per cent.

Having shown the rates at which rateable estates are valued, I shall now proceed (2) to form an Inventory of the estates to be taxed.

Questions.—1. What is the 1st thing requisite in making taxes?—2. What is the 2nd step?

INVENTORY.

Person's names.	Polls, each at $1 ·10 cts.	Oxen, 5 years old, at $0·30 cts.	Oxen, 4 years old, at $0·20 cts.	Cows, 5 years old, at $0·15 cts.	Cattle, 3 years old, at $0·08 cts.	Cattle, 2 years old, at $0·05 cts.			Total amount of rateable estate.
	$1·10	$0·30	$0·20	$0·15	$0·08	$0·05			
A. B.	3	4	2	6	8	8			$6·84
C. D.	1	2	3	4	6	1			$3·43
E. F.	2	3	1	4	2	7			$4·41
G. H.	2			1					$2·35
I. J.	1								$1.10
	9	9	6	15	16	16			$18·13
Proof,	9 polls, each at $1·10 cts.	9 oxen, 5 years old, each at $0·30 cts.	6 oxen, 4 years old, each at $0·20 cts.	15 cows, five years old, each at $0·15 cts.	16 cattle, 3 years old, each at $0·08 cts.	16 cattle, two years old, each at $0·05 cts.			Total amount of rateable estate - -
$18·13	$9·90	$2·70	$1·20	$2·25	$1·28	$0·80			$18·13

NOTE.—I have for brevity sake omitted several articles of rateable estate in the inventory; those which I have inserted I think will sufficiently explain the method of arranging the same.

The method of finding the amount of each man's rateable estate, and of the Inventory.

RULE.—(1) Multiply each man's number of polls by the value of one; his number of oxen by the value of one; his number of cows by the value of one; and so of all the parcels of rateable estate; add the products together, the sum is his rateable estate; (2) add all the rateable estates together, and the sum is the value of the Inventory.

Questions.—1. What is the method or rule of finding the amount of each man's rateable estate?—2. How do you find the amount of the Inventory?

EXAMPLES.

Required the rateable estate of A. B. who has

3	Polls	at $1·10	=	$3·30
4	Oxen five years old	" ·30	=	1·20
2	do four years old	" ·20	=	·40
6	Cows five years old	" ·15	=	·90
8	Cattle three years old	" ·08	=	·64
8	do. two years old	" ·05	=	·40
	Amount of A. B's. rateable estate,			$6·84

In the same manner proceed to find the amount of each man's rateable estate.

To prove the Inventory.

RULE.—(1) Add up the column of polls; multiply the sum by the value of one; the product will be the value of all the polls.

In like manner add every column of the inventory, and multiply the sum of each column by the value of one in in that column: add the several products together, and the sum of all the products will be equal to the sum of all the rateable estates, if the work is right.

Question.—1. How do you prove the Inventory?

EXAMPLES.

The sum of rateable estates in the preceding Inventory is $18.13.

The sum of the products of all the polls and other rateable estates is equal to - - - $18.13 cts.
Therefore the work is right.

(1) *To make the proportion of tax equal, according to the value of each man's rateable estate.*

RULE.—Take tables for every separate tax, (2) by making the sum of the inventory the first term; The sum that is to be raised in any one tax, the second term; and one dollar the third term; the number arising from the operation will be what is raised on one dollar of rateable estate: from this, make (3) the table from 1 to 10, 20, or 30 dollars, as you may think necessary.

(4) In the same manner find what is paid on one cent of rateable estate; and from this, make a table from one to 90 cents; and from these tables take (5) each man's tax.

Questions.—1. After finding the amount of each man's rateable estate and also of Inventory, what is the next step?—2. How do you find what one dollar pays?—3. What do you make from this?—4. How is the cent table found?—5. What do you take from these tables?

EXAMPLES.

1. Suppose the estates named in the preceding inventory are liable to pay $181·30 for the building of a school house; each man's tax is required, in proportion to his rateable estate.

To find what one dollar pays.

If $18·13 cts. pay $181·30 cts., what will 1 dollar pay?

Operation.

D. cts.		*D. cts.*			*D.*
18·13	:	181·30	:	:	1
		1			

18·13)181·30($10 Ans.
181·3
0

To find what 1 cent will pay.

If $18·13 cts. pay $181·30 cts., what will 1 cent pay?

Operation.

D. cts.		*D. cts.*			*cts*
18·13	:	181·30	:	:	01
		·01			

18·13)181·30(10 cents Ans.
181·3
0

TO MAKE THE TABLE.

Explanation.—1 Dollar pays 10 dollars; then 2 dolls. will pay twice as much, which will be 20 dolls.; $3, three times as much, which will be 30 dolls.; and so on to any number you please, always observing to multiply what one dollar pays by the number of dollars.

Or, add what 1 dollar pays with the last found, and you have what the next pays; and so on for the rest.

Dollar Table, from one dollar to eighteen.				Cent Table, from one to ninety cents inclusive.			
1	dollar	pays	$10	1	cent	pays	cts. ·10
2	"	"	20	2	"	"	·20
3	"	"	30	3	"	"	·30
4	"	"	40	4	"	"	·40
5	"	"	50	5	"	"	·50
6	"	"	60	6	"	"	·60
7	"	"	70	7	"	"	·70
8	"	"	80	8	"	"	·80
9	"	"	90	9	"	"	·90
10	"	"	100	10	"	"	1·00
11	"	"	110	20	"	"	2·00
12	"	"	120	30	"	"	3·00
13	"	"	130	40	"	"	4·00
14	"	"	140	50	"	"	5·00
15	"	"	150	60	"	"	6·00
16	"	"	160	70	"	"	7·00
17	"	"	170	80	"	"	8·00
18	"	"	180	90	"	"	9·00

To make the tax by the help of the tables.

1. Required A. B's. tax, his rateable estate being $6·84 cts

$6·00	cents	rateable	estate	pay	$60·00 cts. tax.
·80	"	"	"	pay	8.00
·04	"	"	"	pay	·40
Amount of A. B's. tax					$68·40

·0

2. Required C. D's. tax, his rateable estate being $3·43 cts.

$3·00	cents	rateable	estate	pay	$30·00 cts. tax.
·40	"	"	"	pay	4·00
·03	"	"	"	pay	·30
Amount of C. D's. tax					$34·30

In the same manner take each man's tax from the tables, and their several proportions will be as follows:

FIRST TAX LIST.

A. B.	$68·40
C. D.	$34·30
E. F.	$44·10
G. H.	$23·50
I. J.	$11·00
Am't of tax,	$181·30

PROOF.—(1) Add all the taxes together, the sum must be like the sum raised, if the work is right, except what is lost in fractions. In this example the amount of taxes is just equal to the sum to be raised, therefore the work is right.

Question.—1. How do you prove the tax list?

NOTE.—It must be remembered that no one set of tables will answer for any two different taxes, unless the sum of the two different taxes are alike.

EXAMPLE SECOND.

1. Suppose the estates in the preceding inventory are taxed in a further sum of 19 dollars; I demand the proportion to each man, according to their several rateable estates.

To find what one dollar pays.

Say if 18 dolls. 13 cts. pay 19 dolls., what will 1 doll. pay?

D. cts.		*D.*		*D.*	*D. cts. m*
As 18·13	:	19	: :	1	: 1·04·7 Ans.

To find what one cent will pay.

Say if 18 dolls. 13 cts. pay 19 dolls., what will 1 cent pay?

Operation.

18·13 : 19 00 : : ·01

·01

cts. m

18·13)·1900(·01·04 Ans.

1813

8700

7252

1448

NOTE.—In the above operation, I have carried the quotient into tenths of mills, by the help of which the table is formed; although in the table they are omitted

Dollar Table, from one dollar to eighteen.				Cent Table, from one cent to ninety.			
			D. c. m.				cts.m.
1	Dollar	pays	1·04 7	1	Cent	pays	·01 0
2	"	"	2·09 4	2	"	"	·02 0
3	"	"	3·14 1	3	"	"	·03 1
4	"	"	4·18 8	4	"	"	·04 1
5	"	"	5·23 5	5	"	"	·05 2
6	"	"	6·28 2	6	"	"	·06 2
7	"	"	7·32 9	7	"	"	·07 2
8	"	"	8·37 6	8	"	"	·08 3
9	"	"	9·42 3	9	"	"	·09 3
10	"	"	10·47 0	10	"	"	·10 4
11	"	"	11·51 7	20	"	"	·20 8
12	"	"	12·56 4	30	"	"	·31 2
13	"	"	13·61 1	40	"	"	·41 6
14	"	"	14·65 8	50	"	"	·52 0
15	"	"	15·70 5	60	"	"	·62 4
16	"	"	16·75 2	70	"	"	·72 8
17	"	"	17·79 9	80	"	"	·83 2
18	"	"	18·84 6	90	"	"	·93 6

To make the second tax list.

1. From the last tables it is required to take A. B's. tax. His rateable estate is $6·84 cts.

$6·00	pay	6·28 2
·80	"	·83 2
·04	"	·04 1
Am't. of A. B's tax		$7·15 5

2. From the tables it is required to take C. D's tax. His rateable estate is $8·43 cts.

$8·00 cts.	pay	$8·14 1
·40	"	·41 6
·03	"	·03 1
Am't. of C. D's tax.		$8·58 8

Proceed in like manner to find each man's tax, and their several proportions will stand as in the annexed list.

SECOND TAX LIST.

A. B.	$7·15 5
C. D.	$3·58 8
E. F.	$4·61 4
G. H.	$2·45 8
I. J.	$1·15 1
	$18·96 6

I find by adding up the tax list, that the sum of taxes is 3 cents and 4 mills less than the sum which was to be raised, which was lost in remainders.

NOTE.—In assessing taxes, the law allows the assessor to assess a certain sum, over and above the necessary sum to be raised; or else the tax would not be large enough when there are remainders, which is almost always the case.

INVERSE PROPORTION.

DEFINITION.—Inverse Proportion teaches (1) by having three numbers given to find a fourth, that shall have the same proportion to the third as the first has to the second; (2) when more requires less, or less requires more, the proportion is Inverse; more requires less, (3) when the third term is larger than the first, and the sense of the question requires the fourth to be less than the second; less requires more (4) when the third term is smaller than the first, and the sense of the question requires the fourth term to be greater than the second.

RULE.—(5) Make that number which asks the question the third term; that which is of the same name, the first term; and the other remaining number must be the middle term; (6) multiply the first and second terms together, and divide the product by the third; the quotient will be the answer required.*

Questions.—1. What does Inverse Proportion teach?—2. When is the

*I have followed the same method in *Inverse Proportion* as I did in Direct Proportion: that is, I have done the sums in decimals and then the other way.

proportion Inverse?—3. When does more require less?—4. When does less require more?—5. How do you state the question or arrange the three given numbers?—6. What is the method of operation?

EXAMPLES.

1. If 8 men can build a tower in 12 days, I demand how long it would take them, if 4 more were added to their number?

men.
8
4

men. days.
As 8 : 12 : : 12
8

12)96(8 da. Ans.

NOTE.—In this example it is evident that 12 men would do the work quicker than 8, therefore more requires less: more men, requires less time.

The same question done the other way.

Ans. 8 days.

2. There is a piece of land 16·5 rods long; I demand the width to make an acre, allowing 20 links to a rod.

Ans. 9 696 rods.

The same question done the other way.

Ans. 9 rds. 13† lin.

3. A ship at sea is provided with bread and water sufficient to supply 20 men 9 months, 30 days to a month; but finding a wreck in which were 3 men and 3 boys, to whom they were willing to communicate relief, I demand the time they all can draw their allowance, allowing 2 boys equal to a man.

Ans. 7·346 mo.

The same question done the other way.

Ans. 7 months 10† days

4. There is a piece of land that is 10 rds. 10 links in width; I demand the length to make an acre, allowing 20 links to a rod. Ans. 15·238 rds.

The same question done the other way.

Ans. 15 rods 4† links.

5. How much in length of that which is 5 rods wide will make an acre? Ans. 32 rds.

The same question done the other way.

Ans. 32 rds.

6. How much carpeting that is 1·5 yds. wide will cover a floor that is 7·5 yds. long, and 5 yds. wide?

Ans. 25 yds.

The same question done the other way.

Ans. 25 yds.

7. If 40 men can perform a piece of work in 20 days, how many men can accomplish another piece of work five times as large in a fifth part of the time.

Ans. 1000 men.

8. If in 12 months $100 principal gain $5 interest, what principal will gain the same interest in 5 months?

Ans. $240.

9. How much in length, that is 8 inches wide, will make a square foot? Ans. 4 feet.

DOUBLE RULE OF THREE.

DEFINITION.—The Double Rule of Three teaches (1) by having five numbers given to find a sixth.

RULE.—State the question, (2) by placing the three conditional terms in such order that that number which is the cause of gain, loss, or action, may possess the first place; that which denotes space of time, distance of place, direction or dimension, the second; and that which is the gain, loss, or action, the third: then place the other two terms, which move the question, under those of the same name. Then, if the blank place, or term sought, fall under the third place, the proportion is direct, therefore, (3) multiply the three last terms together, for a dividend, and the other two for a divisor; then the quotient will be the answer; but if the blank fall under the first or second place, the proportion is inverse, wherefore (4) multiply the first, second and last terms together, for a dividend, and the other two, for a divisor: the quotient will be the answer.

NOTE.—The sixth term or answer always will come of the same name, and of the same denomination, of the term directly over the blank place, and may be found by two statements in the Single Rule of Three.

Questions.—1. What does the Double Rule of Three teach?—2. How do you state the question?—3. What is the method of operation, when the

term sought or blank place falls under the 3d term?—4. What, when it falls under the 1st or 2d term?

EXAMPLES.

1. If 200 dollars in 12 months will gain 12 dollars, I demand how many 50 dollars will gain in six months?

dolls. mo. dolls.
200 : 12 : : 12
50 : 6 : :

Three last terms $50\times6\times12=3600$. dividend
Two first terms $200\times12=2400$. divisor.
$3600\div2400=\$1{\cdot}50$ Ans.

2. If 4 men spend 60 dollars, going 300 miles, I demand what is sufficient for the expense of 20 men and one boy 700 miles, allowing the boy one half a man's expense?
Ans. \$717·50.

3. If 3 men can build 360 rods of wall in 24 days, I demand how many rods 7 men would build in 27 days?
Ans. 945 rds.

4. If 3 men in 24 days can build 360 rods of wall, I demand the number of men necessary to build 945 rods in 27 days? Ans. 7 men.

5. If 50 men can build a bridge in 144 days, I demand the number of men necessary to build a like bridge in 720 days? Ans. 10 men.

6. A man lent a friend 600 dollars, for 6 months, for which he received 9 dollars interest; I demand the sum that will gain the same interest in 2 months?
Ans. \$1800.

7. A received of B. 9 dollars, for the loan of 600 dollars six months; now B. wishes to hire of A. 1800 dollars until the loan should amount to the same sum: how long may he keep it? Ans. 2 months.

8. If 6 men build a wall 20 feet long, 6 feet high and 4 feet wide in 16 days, in what time will 24 men build one 200 feet long, 8 feet high and 6 feet thick? Ans. 80 days.

9. If the freight of 12 cwt. 2 qrs. 6 lb. 275 miles, cost \$27·78, how far may 60 cwt. 3 qrs. be shipped for \$234·78?
Ans. 480 miles.

10. If 20 dogs for 30 groats, go 40 weeks to grass;
How many hounds for 60 crowns can winter in that place?
(Call a crown 5s.) Ans. 2000.

VULGAR FRACTIONS.

Definition.—Vulgar Fractions are (1) parts of whole numbers, and are expressed thus, $\frac{1}{2}$, $\frac{3}{7}$, &c. The top figure is (2) a remainder left after division, and is called the numerator: the bottom figure is (3) the divisor used in division, and is called the denominator. Fractions are read thus, $\frac{1}{10}$, is read one tenth; and $\frac{6}{7}$, is read six seventh, &c.

Fractions are (4) proper, improper, compound, or mixed: (5) a Proper Fraction has its numerator the smallest; an Improper Fraction has (6) its numerator equal, or the largest; a Compound Fraction is (7) the fraction of a fraction, and is coupled by the word *of;* a mixed number is (8) a whole number and a fraction.

A proper fraction is written thus, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{8}$, $\frac{9}{12}$, &c.

An improper fraction is written thus, $\frac{7}{4}$, $\frac{5}{5}$, or $\frac{5}{5}$, $\frac{7}{7}$, &c.

A compound fraction is written thus, $\frac{1}{2}$ of $\frac{1}{8}$, of $\frac{1}{4}$ of $\frac{8}{8}$.

A mixed number is written thus, $12\frac{1}{4}$, $6\frac{9}{10}$, $5\frac{4}{5}$, &c.

Questions.—1. What are Vulgar fractions?—2. What is the top figure, and what called?—3. What the bottom figure, and what called?—4. Into what are Vulgar Fractions divided?—5. Describe a Proper Fraction.—6. An Improper.—7. What is a Compound Fraction?—8. What a mixed number?

(2) CASE I.

To find the greatest common measure.

Rule.—(1) Divide the denominator by the numerator, and the last divisor by the last remainder; the last divisor used is the common measure; if 1 is the last divisor, the fraction is in its lowest terms.

Questions.—1. How do you find the greatest common measure?—2. What Case is this?

EXAMPLES.

1. The greatest common measure of $\frac{252}{262}$ is required.

26)252(9
234

18)26(1
18

8)18(2
16

2)8(4
8

The last divisor being 2, it is the greatest common measure of the fraction.

2. What is the greatest common measure of $\frac{964}{1664}$?
Ans. 8.

3. What is the greatest common measure of $\frac{64}{400}$?
Ans. 4.

(2) CASE II.

To reduce fractions to their lowest terms.

RULE.—(1) Find a common measure by case 1st; divide the numerator and denominator of the given fraction, by the common measure, the quotient is the fraction in its lowest terms.

Questions.—1. How do you reduce Fractions to their lowest terms?—2. What Case is this in?

EXAMPLES.

1. Reduce $\frac{322}{464}$ to its lowest terms.
$\frac{322}{464} \div 2$ Com. m.$=\frac{161}{232}$ Ans.

2. It is required to reduce $\frac{964}{1664}$ to its lowest terms.
Ans. $\frac{108}{133}$.

3. Required to reduce $\frac{64}{400}$ to its lowest terms.
Ans. $\frac{16}{115}$.

CASE III.

To reduce a mixed number to an improper fraction.

RULE.—(1) Multiply the whole number by the denominator of the fraction, to the product add the numerator of the fraction; this sum placed over the given denominator will form the fraction required.

Question.—1. How do you reduce a mixed number to an improper fraction?

EXAMPLES.

1. Reduce $10\frac{3}{4}$ to an improper fraction.
$10\times4=40+3=43$ numerator, $\frac{43}{4}$ Ans.

2. Reduce $112\frac{1}{3}$ to an improper fraction.
$112\times3+1=\frac{337}{3}$ Ans.

3. Reduce $28\frac{9}{10}$ to an improper fraction. Ans. $\frac{289}{10}$.

CASE IV.

To reduce an improper fraction to its proper terms.

RULE.—(1) Divide the numerator by the denominator,

the quotient is the whole number; the remainder is the numerator of the fraction.

Question.—1. How do you reduce an improper fraction to its proper terms?

EXAMPLES.

1. Reduce $\frac{43}{4}$ to its proper terms. $43 \div 4 = 10\frac{3}{4}$ Ans
2. Reduce $\frac{339}{3}$ to its proper terms. Ans. $113\frac{0}{3}$.
3. Reduce $\frac{289}{10}$ to its proper terms. Ans. $28\frac{9}{10}$.

CASE. V.

To find the least common denominator.

RULE.—(1) Divide the given denominators by any number that will divide two or more of them without a remainder; then (2) divide the undivided numbers and last quotients, by any number that will divide two or more of them without a remainder; and thus continue dividing (3) till no two numbers can be divided without a remainder; then (4) multiply the divisors and remainders together, the product is the common denominator required.

Questions.—1. What is the first step in finding the least common denominator?—2. What is the 2d step?—3. How long do you thus continue to divide?—4. What is the last step?

EXAMPLES.

1. What is the least common denominator of $\frac{4}{6}$, $\frac{1}{8}$, $\frac{4}{7}$, $\frac{3}{10}$, $\frac{9}{18}$?

3 | 6 8 7 10 18 given denominators.

2 | 2 8 7 10 6

com. denom. $1 \times 4 \times 7 \times 5 \times 3 \times 3 \times 2 = 2520$ Ans.

2. What is the common denominator of $\frac{3}{5}$, $\frac{6}{14}$, $\frac{4}{7}$, $\frac{12}{16}$, and $\frac{3}{4}$? Ans. 1008.

CASE VI.

To reduce fractions to a common denominator.

RULE.—(1) Find the common denominator by Case 5th, and use it as a dividend, and use each given denominator as a divisor; (2) divide; (3) multiply the quotients by the respective numerators, and the product placed over the common denominator will form the fractions required.

Questions.—1. What is the 1st step to reduce fractions to a common denominator?—2. What the 2d step?—3. What the 3d and last?

EXAMPLES.

1. Reduce $\frac{3}{4}$, $\frac{2}{5}$, $\frac{7}{8}$, $\frac{9}{10}$ to common denominators.

Com. denom. $40 \div 4 = 10 \times 3 = 30$ 1st numerator.
$40 \div 5 = 8 \times 2 = 16$ 2d numerator.
$40 \div 8 = 5 \times 7 = 35$ 3d numerator.
$40 \div 10 = 4 \times 9 = 36$ 4th numerator.

The new numerators being written over the common denominator, the fractions appear thus, $\frac{30}{40}$, $\frac{16}{40}$, $\frac{35}{40}$, $\frac{36}{40}$ Ans.

2. Reduce $\frac{2}{5}$, $\frac{3}{10}$, $\frac{6}{14}$, $\frac{11}{16}$, and $\frac{4}{8}$ to common denominators.

Ans. $\frac{560}{1680}$, $\frac{1344}{1680}$, $\frac{720}{1680}$, $\frac{840}{1680}$, $\frac{1120}{1680}$.

3. Reduce $\frac{4}{6}$, $\frac{8}{10}$, $\frac{9}{12}$, and $\frac{11}{14}$, to common denominators. Ans. $\frac{280}{420}$, $\frac{336}{420}$, $\frac{315}{420}$, $\frac{330}{420}$.

CASE VII.

To reduce compound fractions to simple ones.

RULE.—(1) Multiply all the numerators together for a new numerator; and all the denominators together for a new denominator, and it is done.

Question.—1. How do you reduce compound fractions to simple ones?

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{4}{7}$ of $\frac{6}{8}$ to a simple fraction.

Num. $1 \times 3 \times 5 \times 4 \times 6 = 360$
— — — — — — Ans.
Den. $2 \times 4 \times 6 \times 7 \times 8 = 2688$

2. Reduce $\frac{9}{10}$ of $\frac{1}{2}$ of $\frac{3}{6}$ of $\frac{1}{12}$ of $\frac{9}{10}$ to a simple fraction, and to its lowest terms. Ans. $\frac{27}{1440}$.

CASE VIII.

To reduce a fraction of a lower denomination to the fraction of a higher, retaining the same value.

RULE.—(1) Reduce the fraction to a compound one, by comparing it with all the denominations between itself and the denomination to which you would reduce it; then reduce it to a simple fraction by Case 7th.*

Question.—1. How do you reduce a fraction of a lower denomination to that of a higher, retaining the same value?

* NOTE.—When either of the former Cases is requisite for the reduction of fractions, the scholar should be particularly required to give a full and explicit explanation of that Case, as for instance; the last step in Case 8th is to reduce the fraction to a simple one by Case 7th. Now here the question should occur; What does Case 7th require?—And the answer to the 1st question under that Case will be a sufficient explanation.—See also Cases 2d and 6th.

EXAMPLES.

1. Reduce $\frac{3}{10}$ of a mill to the fraction of a dollar.

By comparing it, it becomes $\frac{3}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$.

Ans. $\frac{3}{10000}$.

2. Reduce $\frac{1}{2}$ of a grain to the fraction of a pound Troy weight. $\frac{1}{2}$ of $\frac{1}{24}$ of $\frac{1}{20}$ of $\frac{1}{12}$ of $\frac{1}{1}=\frac{1}{5760}$ Ans.

3. Reduce $\frac{3}{4}$ of a day to the fraction of a year containing 365 days. Ans. $\frac{3}{1460}$.

RULE 2.—(1) Reduce the Integer to the same denomination with the denominator of the fraction, for a denominator, over which place the numerator of the given fraction, and it will form the fraction required.

Questions.—1. By rule 2d, how do you reduce a fraction of a lower denomination to that of a higher, retaining the same value?

EXAMPLES.

1. Required to reduce $\frac{3}{10}$ of a mill, to the fraction of a dollar.

dolls. d. cts. m. tenths.

$1\times10\times10\times10\times10=10000$ denominator.

Ans. $\frac{3}{10000}$.

2. Reduce $\frac{3}{4}$ of a day to the fraction of a year.

Ans. $\frac{3}{1460}$.

CASE IX.

To reduce the fractions of a higher, to the fraction of a lower denomination, retaining the same value.

RULE.—(1) Multiply the numerator of the given fraction into the denominators of the compound ones between itself and the denomination to which you would reduce it, for a new numerator; which being placed over the given denominator, will form the fraction required.

RULE 2.—(2) Invert the denominations between the given denomination, and the one to which you would reduce it; then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

Questions.—1. How do you reduce fractions of a higher denomination to that of a lower, retaining the same value? 2. What method does rule second give?

EXAMPLES.

1. Reduce $\frac{3}{1000}$ of a dollar, to the fraction of a mill.

D. d. cts. m.

$\frac{3}{1000} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{300}{1000}$ given Denom. Ans.

2. Reduce $\frac{3}{1460}$ of a year to the fraction of a day.

$\frac{3}{1460}$ of $\frac{1}{3}\frac{3}{65}$ inverted is $\frac{3}{1460}$ of $\frac{1}{1}$ of $\frac{365}{1}$

and $\frac{3}{1460} \times \frac{1}{1} \times \frac{365}{1} = \frac{1095}{1460}$ Ans

CASE X.

To express the value of a fraction in the several denominations of the Integer.

RULE.—(1) Multiply the numerator of the fraction into the known denominations of the Integer, and divide the product by the denominator, the quotient is the value of the fraction, in the deominations of the Integer.

Question.—1. How do you express the value of a fraction in the several denominations of the Integer?

EXAMPLES.

1. What is the value of $\frac{9}{10}$ of a dollar?

1 Doll. $= 100 \times 9 = 900 \div 10 = 90$ cts. Ans.

2. What is the value of $\frac{1}{6}$ of an cwt. avoirdupois?

Ans. 2 qr. 18 lb. 10 oz. $10\frac{2}{3}$ dr.

3. What is the value of $\frac{8}{12}$ of a cord of wood?

Ans. 96 solid feet.

4. What is the value of $\frac{9}{10}$ of a yard?

Ans. 3 qr. $2\frac{4}{10}$ na.

CASE XI.

To reduce parts of an Integer which are expressed in several denominations, to the fraction of a greater of the same kind.

RULE.—(1) Reduce the several denominations to the lowest denomination mentioned for a numerator; and reduce the integer to the same denomination for a denominator.

Question.—1. How do you reduce parts of an Integer, expressed in several denominations, to the fraction of a greater of the same kind?

EXAMPLES.

1. It is required to reduce 1 dime, 6 cents, and 9 m. to the fraction of an eagle.

1 dime 6 cts. 9m. $= \frac{144}{1000}$ Ans.
1 eagle.

2. Reduce 2 rods 12 feet to the fraction of a mile and to its lowest terms. Ans. $\frac{1}{220}$.

3. Reduce 10 feet 100 inches to the fraction of a cord, wood measure. Ans. $\frac{175}{2112}$.

NOTE.—Before the scholar proceeds further, let the Instructer examine him thoroughly in the several Cases in Reduction promiscuously; for this is certain that, when once these Cases are familiar, all difficulties in Vulgar Fractions are in a measure surmounted.

ADDITION OF VULGAR FRACTIONS.

CASE I.

To add fractions having common denominators.

RULE.—(1) Add the numerators all together, divide the sum by the common denominator, the quotient is the answer.

Question.—1. How do you add fractions having common denominators?

EXAMPLES.

Add $\frac{9}{15}$, $\frac{10}{15}$, $\frac{6}{15}$, $\frac{5}{15}$, $\frac{11}{15}$, $\frac{12}{15}$, $\frac{14}{15}$ together.

The numerators $9+10+6+5+11+12+14=67$

$67 \div 15 = 4\frac{7}{15}$ Ans.

CASE II.

To add fractions having different denominators.

RULE.—(1) Find a common denominator by Case 5th* (in reduction,) use it as a dividend; use each given denominator as a divisor; divide, and multiply the quotients by

* The remark made in Note under Case 8 in Reduction of Fractions should here be observed, as also in many subsequent Cases.

the several numerators, and the sum of all the products is the numerator to the common denominator, and is the fraction required.

Question.—1. How do you add fractions having different denominators?

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{6}$, $\frac{6}{8}$, $\frac{8}{10}$, $\frac{10}{12}$, $\frac{12}{14}$ together.

	denom.	*numer.*
Com. Den.	$840 \div 2 = 420$	$\times 1 = 420$
	$840 \div 5 = 168$	$\times 3 = 504$
	$840 \div 6 = 140$	$\times 4 = 560$
	$840 \div 8 = 105$	$\times 6 = 630$
	$840 \div 10 = 84$	$\times 8 = 672$
	$840 \div 12 = 70$	$\times 10 = 700$
	$840 \div 14 = 60$	$\times 12 = 720$

numerator 4206 $\frac{4206}{840}$ Ans.

2. It is required to add $\frac{9}{10}$, $\frac{5}{8}$, $\frac{12}{14}$ together.

Ans. $2\frac{142}{280}$.

3. It is required to add $\frac{1}{4}$, $\frac{6}{7}$, $\frac{8}{9}$, $\frac{10}{11}$, $\frac{11}{12}$ together.

Ans. $4\frac{1398}{2772}$.

CASE III.

To add mixed numbers whose fractions have common denominators.

RULE.—(1) Add the fractions as in Case 1st, and the whole numbers as in Simple Addition; then add the sum of the fractions to the whole number.

Question.—1. How do you add mixed numbers whose fractions have common denominators?

EXAMPLES.

1. Required to add $104\frac{3}{20}$, $111\frac{16}{20}$, $96\frac{14}{20}$, $80\frac{16}{20}$.

whole num. $104+111+96+80=391$

num of frac. $2+16+14+16=48 \div 20= \quad 2\frac{8}{20}$.

Ans. $393\frac{8}{20}$.

2. Required to add $14\frac{6}{13}$, $15\frac{2}{13}$, $18\frac{9}{13}$, $7\frac{10}{13}$, together.

Ans. $56\frac{1}{13}$.

CASE IV.

To add mixed numbers whose fractions have different denominators.

RULE.—(1) Add the fractions as in Case second, and the whole numbers as in Simple Addition; then add the sum of the fractions to the sum of the whole numbers, and it is done.

Question.—1. How do you add mixed numbers, whose fractions have different denominators?

EXAMPLES.

1. Add $12\frac{1}{2}$, $16\frac{2}{3}$, $17\frac{3}{4}$, $16\frac{1}{12}$, and $19\frac{7}{8}$ together.

Whole num. 12+16+17+16+19=80

Sum of the fractions $3\frac{5}{24}$

Ans. $83\frac{5}{24}$

2. Add $21\frac{9}{10}$, $6\frac{7}{12}$, $127\frac{1}{2}$, and $14121\frac{1}{3}$ together.

Ans. $14277\frac{7}{120}$

CASE V.

To add fractions of several denominations, such as parts of cwts., parts of qrs., parts of lbs., &c.

RULE.—(1) Find the value of each separate fraction by Case tenth in Reduction, and add the several sums together as in Compound Addition.

Question.—1. How do you add fractions of several denominations, such as parts of cwts., parts of qrs., &c.?

EXAMPLES.

1. Add $\frac{1}{2}$ of a cwt. $\frac{3}{4}$ of a qr. and $\frac{4}{5}$ of a lb. together.

	qr.	*lb.*	*oz.*	*dr.*
$\frac{1}{2}$ of a cwt. is	2	0	0	0
$\frac{3}{4}$ of a qr. is	0	21	0	0
$\frac{4}{5}$ of a lb. is	0	00	13	$5\frac{3}{5}$
	2	21	13	$5\frac{3}{5}$ Ans.

2. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour and $\frac{3}{4}$ of a minute, together.

Ans. 2 days, 2 hours, 30 minutes, 45 seconds.

SUBTRACTION OF VULGAR FRACTIONS.

CASE I.

To subtract simple fractions having common denominators.

RULE.—(1) Subtract the least numerator from the largest, and the remainder placed over the common denominator will be the difference required.

Question.—1. How do you subtract simple fractions having common denominators.

EXAMPLES.

1. What is the difference between $\frac{12}{19}$ and $\frac{10}{19}$?
   largest numerator 12
   smallest numerator 10

   2 difference $=\frac{2}{19}$ Ans.
2. What is the difference between $\frac{22}{41}$ and $\frac{15}{41}$?
   Ans. $\frac{7}{41}$.
3. What is the difference between $\frac{96}{212}$ and $\frac{74}{212}$?
   Ans. $\frac{22}{212}$.

CASE II.

To subtract a simple fraction, or a mixed number from a whole number.

RULE.—(1) Subtract the numerator of the fraction from its denominator, and put the denominator of the fraction under the remainder; and carry one to be deducted from the whole number.

Question.—1. How do you subtract a simple fraction, or a mixed number, from a whole number?

EXAMPLES.

1. It is required to subtract $\frac{3}{5}$ from 12.
   $12-\frac{3}{5}=11\frac{4}{5}$ Ans.
2. It is required to subtract $27\frac{48}{97}$ from 32.
   Ans. $4\frac{48}{97}$.
3. What is the difference between $12\frac{9}{10}$ and 21.
   Ans. $8\frac{1}{10}$.
4. What is the difference between 101 and $100\frac{99}{100}$.
   Ans. $\frac{1}{100}$.

CASE III.

To subtract fractions having different denominators.

RULE.—(1) Reduce the fractions to a common denominator, by Case 5th in Reduction; and the difference of the numerators is the difference required.

Question.—1. How do you subtract fractions having different denominators?

EXAMPLES.

1. What is the difference between $\frac{1}{2}\frac{8}{0}$ and $\frac{2}{4}\frac{3}{0}$?

$\frac{1}{2}\frac{8}{0}$ and $\frac{2}{4}\frac{3}{0}$ are equal to $\frac{1}{1}\frac{7}{0}\frac{1}{0}$, $\frac{1}{1}\frac{8}{0}$ difference, $\frac{8}{1}\frac{3}{8}\frac{3}{0}$ Ans.

2. What is the difference between $\frac{1}{1}\frac{1}{2}$ and $\frac{9}{1}\frac{9}{0}$?

Ans. $\frac{1}{3}\frac{1}{0}$.

CASE IV.

To distinguish the largest of any two fractions, &c.

RULE.—(1) Reduce them to a common denominator, (by Case 5th in Reduction,) and the one that has the largest numerator is the largest fraction.

Question.—1. What method do you take to distinguish the largest of any two fractions?

EXAMPLES.

1. Which is the greatest fraction $\frac{9}{1}\frac{9}{0}$ or $\frac{1}{1}\frac{1}{2}$

$\frac{1}{1}\frac{1}{2}$ reduced to a common denom. is equal to $\frac{6}{8}\frac{5}{0}$

$\frac{9}{1}\frac{9}{0}$ reduced to a common denom. is equal to $\frac{6}{8}\frac{4}{0}$

Therefore $\frac{1}{1}\frac{1}{2}$ is the largest fraction by $\frac{1}{8}\frac{1}{0}$.

2. Which is the greatest fraction $\frac{2}{5}\frac{3}{3}$ or $\frac{3}{7}$, and by how much? Ans. $\frac{2}{5}\frac{3}{3}$ is the largest by $\frac{1}{3}$.

CASE V.

To subtract a mixed number from a mixed number, when the fraction which is to be subtracted is greater than the one which it is to be taken from.

RULE.—(1) Reduce the fractions to a common denominator, then subtract the numerator of the greater from the common denominator, and to the remainder add the least numerator; and the sum of them must be set over the com-

mon denominator; and carry one to the whole number, and subtract the whole number as in Simple Subtraction.

Question.—1. How do you subtract a mixed number from a mixed number, when the fraction which is to be subtracted is greater than the one from which it is to be taken?

EXAMPLES.

1. It is required to take $8\frac{1}{1}\frac{3}{6}$ from $12\frac{3}{8}$.

$\frac{1}{1}\frac{3}{6}$ reduced to a common denom. is equal to $\frac{7}{1}\frac{3}{1}\frac{2}{4}$.

$\frac{3}{8}$ reduced to a common denom. is equal to $\frac{5}{1}\frac{7}{1}\frac{7}{4}$.

$12\frac{5}{1}\frac{7}{1}\frac{7}{4}$ less by $8\frac{7}{1}\frac{3}{1}\frac{2}{4}=3\frac{9}{1}\frac{9}{1}\frac{5}{4}$ Ans.

2. It is required to take $9\frac{1}{1}\frac{5}{6}$ from $10\frac{1}{2}$.

Ans. $\frac{1}{1}\frac{1}{6}$.

3. It is required to take $1\frac{7}{1}\frac{7}{2}$ from $10\frac{3}{1}\frac{3}{0}$.

Ans. $8\frac{1}{6}\frac{3}{0}$.

CASE VI.

To subtract when the fractions are of different denominations; such as parts of cwt. qr. lb. &c.

RULE.—(1) Find the value of the fraction (by case 10th in Reduction,) then subtract as in Compound Subtraction. If fractions happen in finding the value of the parts, (2) reduce them to common denominators and subtract as in Case third.

Questions.—1. How do you subtract when the fractions are of different denominations; such as parts of cwt. qr. lb. &c.?—2. If fractions happen in finding the value of the parts, what do you do?

EXAMPLES.

1. It is required to take $\frac{9}{1}\frac{}{0}$ of a cwt. from $\frac{4}{5}$ of a ton

	cwt.	*qr.*	*lb.*	*oz.*	*dr.*
$\frac{4}{5}$ of a ton =	15	0	0	0	0
$\frac{9}{1}\frac{}{0}$ of a cwt. =	0	3	16	12	$12\frac{8}{1}\frac{}{0}$
	14	0	11	3	$3\frac{3}{1}\frac{}{0}$ Ans.

2. Required to take $\frac{6}{1}\frac{}{8}$ of a cwt. from $12\frac{1}{2}$ cwt.

Ans. 12 cwt. 0 qrs. 18 lb. 10 oz. $10\frac{1}{1}\frac{2}{8}$ dr.

MULTIPLICATION OF VULGAR FRACTIONS.

CASE I.

RULE.—(1) Reduce compound fractions to simple ones; mixed numbers to improper fractions; then multiply the numerators all together for a new numerator, and the denominators for a new denominator.

Question.—1. What is the general rule for multiplying Vulgar Fractions?

EXAMPLES.

1. It is required to multiply $\frac{3}{8}$ by $\frac{3}{16}$ of $\frac{4}{5}$ of $\frac{1}{4}$.

$\frac{3}{16}$ of $\frac{4}{5}$ of $\frac{1}{4}=\frac{12}{320}\times\frac{3}{1}=\frac{36}{320}=\frac{9}{80}$ Ans.

2. It is required to multiply $2\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{8}{9}$.

Ans. $\frac{8}{9}\frac{8}{4}=1\frac{2}{9}$.

3. It is required to multiply $\frac{4}{12}$ by $\frac{9}{16}$.

Ans. $\frac{4}{12}\frac{9}{16}=\frac{3}{16}$.

4. It is required to multiply $\frac{1}{2}$ by $\frac{1}{3}$. Ans. $\frac{1}{6}$.

DIVISION OF VULGAR FRACTIONS.

RULE.—(1) Reduce compound fractions to simple ones: mixed numbers to improper fractions; then invert the divisor, (that is, put the numerator for the denominator) and multiply the numerators together for a new numerator, and the denominators together for a new denominator; and the fraction thus found is the quotient required.

Question.—1. What is the general rule for Division of Vulgar Fractions?

EXAMPLES.

1. It is required to divide $\frac{1}{3}$ by $\frac{3}{8}$.

$\frac{3}{8}$ inverted is $\frac{3}{8}\times\frac{1}{3}=\frac{3}{24}$ Ans.

2. It is required to divide $\frac{4}{7}$ by $\frac{7}{16}$. Ans. $7\frac{1}{7}$.

3. It is required to divide 10 by $\frac{1}{2}$ of $\frac{3}{5}$. Ans. $33\frac{1}{3}$.

4. It is required to divide $10\frac{4}{7}$ by $7\frac{3}{16}$. Ans. $1\frac{4}{7}\frac{9}{11}$.

Miscellaneous Questions in Vulgar Fractions.

1. A school master being asked one day how many scholars he had, made answer that one fifth of them sat on one

seat, one tenth on another, two fifths on another, two tenths on another, and 6 on another; I demand the number his school consisted of.

$\frac{1}{2}+\frac{1}{10}+\frac{2}{5}+\frac{2}{10}=\frac{9}{10}$ 10—9=1=$\frac{10}{1}$= of $\frac{9}{10}$ of $\frac{6}{1}$=$\frac{54}{10}$= 54+6=60. Ans.

2. A man having returned from a journey, was asked by a friend, what his expenses were; the answer was, that he spent in Boston $\frac{2}{5}$ of his money; between Boston and New-York $\frac{1}{2}$; in New-York $\frac{1}{10}$; beyond New-York $\frac{1}{20}$; and he brought home $40; I demand the sum he carried, and also what he spent.

Ans. carried $160, and spent $120.

RULE OF THREE IN VULGAR FRACTIONS.

Rule.—State the question (1) as in the Rule of Three Direct, (2) invert the divisor, and multiply the numerators together, and the denominators together, and the fraction thus found is the fourth term or answer; (3) compound fractions must be reduced to simple ones, and mixed numbers to improper fractions.

Questions.—1. How do you state the question in the Rule of Three in Vulgar Fractions?—2. How then do you proceed?—3. How are Compound Fractions and mixed numbers prepared for the operation?

EXAMPLES.

1. If $\frac{1}{2}$ bushel of corn cost $\frac{75}{100}$ of a dollar, what will $\frac{3}{4}$ of a bushel come to?

As $\frac{1}{2}$: $\frac{75}{100}$: : $\frac{3}{4}$ Dolls.

$\frac{1}{2}$ inverted is $\frac{2\times 75\times 3}{1\times 100\times 4}=\frac{450}{400}=1\frac{1}{8}$ Ans.

2. If $10\frac{1}{4}$ bushels of potatoes cost $\$3\frac{1}{4}$, how many may be had for $\$12\frac{1}{2}$? Ans. $4\frac{100}{164}=39\frac{44}{164}$.

3. If $\frac{1}{4}$ of $\frac{7}{8}$ of a ship is worth $\frac{2}{3}$ of 16000 dollars, how much is $\frac{1}{2}$ of $\frac{1}{5}$ of her worth?

Ans. $\$4571\cdot 42\cdot 8\frac{3}{7}\frac{4}{7}$.

SIMPLE INTEREST.

DEFINITION.—Interest is (1) a premium paid to the lender, by the borrower, for the use of money lent. Simple Interest is reckoned (2) at 6 per cent. that is, at the rate of 6 dolls. for the use of 100 dolls. one year; and so in proportion to a longer, or shorter time, or for a smaller or larger sum.

Principal is (3) the money lent.

Rate per cent. is (4) the price agreed on.

The amount is (5) the principal and interest together.

Questions.—1. What is Interest?—2. How is Simple Interest reckoned?—3. What is principal?—4. What is rate per cent?—5. What is the amount?

CASE I.

The principal and rate given to find the interest.

RULE.—(1) Multiply the principal by the rate per cent and that product by the number of years; the last product pointed, according to the rule of decimals, will be the answer for that time, in dollars and parts of a dollar.

Question.—1. When the principal and rate per cent. are given, how do you find the interest?

EXAMPLES.

1. What is the interest of $22·16 cents for one year, at 6 per cent. per annum?

dolls.cts.	
22·16	principal.
·06*	rate per cent.
1·3296	
1	time.

$1·3296=$1·32·9† Ans.

2. What is the interest of $223.14 cts. for 5 years, at 6 per cent. per annum?

yrs.	*D. cts. m.*
$223.14×·06=13·38	84×5=66·94·2† Ans.

3. What is the interest of 123 dols. 10 cts. 1 m. for 7 years, at 9 per cent. per annum? Ans. $77·55·3·†

* Six per cent. must be written ·06 and seven per cent. ·07, &c. this being observed, the rule of decimal fractions exactly applies.

CASE II.

When the amount is required.

RULE.—(1)Find the interest as in case first, and to it add the principal, the sum is the amount.

Question.—1. How do you find the amount?

EXAMPLES.

1. What will $10.15 cts. amount to in 12 years, at 6 per cent. per annum.?

$10·15
·06

·6090
12

7.3080 interest.
10.15 principal added.

Ans. $17·4580 amount.

2. What is the amount of $1·12 cts. for 12 years, at 12 per cent. per annum? Ans. $2·73·2.†

3. What is the amount of $242.17 cts. for 3 years, at 9 per cent. per annum? Ans. $307·55·5.†

CASE III.

When there are years, months and days in the time.

RULE 1st.—(1) Reduce the days and months to the decimal of a year, then multiply the principal, rate, and time together, the product pointed for decimals according to rule will be the answer.

RULE 2d.—(2) Find the interest for the years by case first; for the months, divide a year's interest by the aliquot parts of a year; and for the days, divide a month's interest by the aliquot parts of a month; the several quotients added with the interest for the years will be the answer.

Questions.—1. What is your 1st rule to find the interest, when there are years, months, and days in the time?—2. What is your 2d rule?

EXAMPLES.

1. What is the interest of $6·22 cts. for 2 yrs. 6 mo. 10 days, at 6 per cent. per annum?

years mo. days yrs. D. cts. cts. m.
2 6 10=2·527 × 6·22 × ·06=94·3† Ans.

The same question by rule second.

$6·22 principal.
·06 rate.

6 mo.=½ of a yr.½)·3732 1 year's interest.
2

·7464 2 year's interest.
·1866 6 month's interest.
·0103† 10 day's interest.

Ans. ·9433† interest for 2 yrs. 6 m. 10 d.

2. What is the interest of $10·10 cts. for 10 years, 11 months, at 9 per cent.?

years	*mo.*	*years*	*D. cts.*		*c. m.*
10	11=10·916	×	10·10	× ·09 =	$9·92·2† Ans.

The same question by rule second.

10·10
·09

6 mo.=½ of 1 year. ½) ·9090 1 year's interest.
10

9·0900 10 year's interest.
3=½ of 6 mo. ½) 0·4545 6 month's interest
2=⅓ of 6 " ⅓) 0·2272† 3 " "
0·1515 2 " "

D.cts.m.
$9·9232† Ans. 9·92·3†

3. What is the interest of $213·23 cts. for 3 years, 1 days, at 10 per cent.? Ans. $64·67 cts. 9†m.

The same question by rule second.

Ans. $64·67 cts. 9†m.

4. What is the interest of $23·23 cts. for 1 year, 9 mo. a 6 per cent.? Ans. $2·43 cts. 9†m.

The same question by rule second.

Ans. $2·43 cts. 9†m.

5. What is the interest of $121·11 cts. for 2 years, 7 mo at 5 per cent.? Ans. $15·64 cts. 3†m.

The same question by rule second.

Ans. $15·64 cts. 3†m.

NOTE.—In the preceding examples I have considered 30 days a month, and 12 months a year; which is the gen-

eral method of casting interest; but by this method 5 days are lost in a year; 12×30=360 days; which in large sums would amount to something worthy of notice.

CASE IV.

(1) *To cast interest for any number of days; allowing three hundred sixty-five days to a year.*

RULE.—(2) Reduce the given number of days to the decimal of a year, which contains 365 days; then (3) multiply the principal, rate, and time together, point off for decimals according to the rule of decimal fractions, and the product is the answer.

Questions.—1. What does Case 4th in Simple Interest teach?—2. What is the 1st step in the rule?—3. What is the 2d step?

EXAMPLES.

1. What is the interest of $21·20 cts. for 27 days, at 9 per cent. per annum?

27 days is $\frac{27}{365}$=·0739

$21·20×·09×·0739=·14 cts. 1† mill. Ans.

2. What is the interest of $12·06, for 3 yrs. 141 days at 6 per cent. per annum? Ans. $2·45†.

3. What is the interest of $3·03, for 136 days, at 6 per cent. per annum? Ans. ·06 cts. 7†m.

CASE V.

When the time is months and the rate per cent. is six.

RULE.—(1) Multiply the principal by half the number of months, (which is just equal to the rate for the time when the annual rate per cent. is six) if the months are odd, annex ·5 to the right hand, the product is the interest for the time.

Question.—1. What is the method of casting interest when the time is months, and rate per cent. 6?

EXAMPLES.

1. What is the interest of $21 for 10 months, at 6 per cent. per annum?

½)10 months =·05×$21=1·05 cts. Ans.

2. What is the interest of 250 dollars 11 cents, for 16 months, at 6 per cent. per annum?

Ans. $20·00 cts. 8†m.

3. What is the interest of $121·12 cents, for 21 months, at 6 per cent. per annum? Ans. $12·71 cts. 7†m.

4. What is the interest of 9 dollars, 9 cents, for 9 months, at 6 per cent. per annum? Ans. 40 cts. 9†m.

CASE VI.

When the time is months and days, and the rate per cent. is 6.

RULE.—(1) Multiply the principal by ½ the number of months and ⅙ of the days (calling ½ the months so many cents and ⅙ of the days so many mills, which is just equal to the rate per cent, for the time,) and the product pointed according to the rule of Decimal Fractions will be the interest required. If there be an odd month, (2) call it 30 days, to which add the given days, if any, and take ⅙ of the whole, which use in the place of mills in the multiplier: Should there be a remainder in taking ⅙ of the days, (3) use it as a fraction, for which take aliquot parts of the multiplicand, which must be added to the product of the *principal* multipled by ½ the number of months, &c. If the number of days be less than 6, so that 6 cannot be contained in them, (4) put a cypher in the place of mills at the right hand of months, and make a fraction of the days, then proceed as before.

NOTE.—If the time is years, months and days, (5) reduce the years to months, to which add the given months and then proceed as before.

Questions.—1. What is the method of casting interest, when the time is months and days, and the rate per cent. 6?—2. What must be done when there is an odd month?—3. What should be done when there is a remainder in taking ⅙ of the days?—4. What method do you take, when the number of days are less than 6?—5. Should there be years in the time, what method is used?

EXAMPLES.

1. What is the interest of $14·24 for 11 months and 24 days at 6 per cent.? Ans. $0·84·0m.

Operation.

½)11 months 24 days.

05 and 1 over=30 " odd month

½)54

9

—$14·24×·059=$0·84·0 m. Ans.

2. What is the interest of $24·17 for 11 months and 29 days at 6 per cent.? Ans. $1·44·6.

3. What is the interest of $35 for 7 months and 19 days at 6 per cent.? Ans. $1·38·5.

4. What is the interest of $75 for 4 months and 2 days at 6 per cent.? Ans. $1·52·5.

5. A note for $27·55 on interest, was dated Feb. 14, 1823; what was the amount Jan. 20, 1825? Ans. $30·74·5m.

CASE VII.

When the rate is any other than 6 per cent.

RULE.—(1) Find the interest at 6 per cent. by Case 5th or 6th, and for the per cent. above or below 6, add or subtract as the case may be. Thus, if it be required to find the interest of a certain sum at 7 per cent.; (2) first find the interest at 6 per cent. and add $\frac{1}{6}$ of this interest to itself (which is equal to the 1 per cent.) and the sum will be the interest at 7 per cent; and (3) for 8 per cent. add $\frac{1}{3}$, for 9 add $\frac{1}{2}$, for 10 add $\frac{2}{3}$, for 11 add $\frac{5}{6}$, and for 12 double the interest at 6 per cent.

So also if the per cent. be less than 6; (4) first find the interest at 6, then subtract $\frac{1}{6}$ of this interest from itself, the remainder is the interest at 5 per cent. subtract $\frac{1}{5}$ and the remainder will be the interest at 4 per cent., and for 3 per cent. subtract $\frac{1}{4}$ and so on to any less per cent.; and it will give the interest required.

Questions.—1. What is the method of casting interest when the rate is any other than 6?—2. How do you proceed when interest at 7 per cent. is required?—3. How do you proceed when 8, 9, 10, 11, or 12 per cent. is required?—4. If the per cent. be less than 6, what is your method?

EXAMPLES.

1. What is the interest of $34·24 for 10 months and 12 days at 7 per cent. per annum? Ans. $2·07 7.

$\frac{1}{2}$)10 months $\frac{1}{6}$)12 days.

·05 2

$34·24
·0 52

6 8 48
1 71 2 0

$\frac{1}{6}$)1·78·0 48=interest at 6 per cent.
29 6 74= " " 1 per cent.

$2·07·7 22=interest at 7 per cent.

2. What is the interest of $13·41 for 3 months and 16 days at 8 per cent.? Ans. $0·31·4.

3. What is the interest of $49·25 for 3 years, 3 months and 3 days at 9 per cent.? Ans. $14·44·2.

4. What is the amount of $5·98 for two years and 8 months at 3 per cent.? Ans. $6·45·8.

5. A note on interest for $45·45, was dated April 26, 1832; what was the amount May 28, 1833 at 4 per cent. per annum? Ans. $47·43.

6. What is the interest of $19 for 22 months, at 12 per cent. per annum? Ans. $4·18 cents

CASE. VIII.

An easy method of casting Interest for any number of days allowing 360 days for a year.

Rule.—(1) Divide 360 by the rate per cent. and the quotient will be a fixed divisor for that per cent.—(2) Multiply the principal by the given number of days in the time, and divide the product by the fixed divisor and the quotient will be the interest in cents. If dollars and cents are named in the sum (3) multiply and divide as before, and cut off two figures from the right of the quotient and the remaining figures on the left will be cents.

Questions.—1. What is the 1st step in Case 8th in casting interest for any number of days?—2. How then do you proceed?—3. If dollars and cents are named in the sum what do you do?

EXAMPLES.

1. What is the interest of $72 for 90 days, at 6 per cent. per annum?

6)360

60 fixed divisor. 72×90=6400÷60=108 cts. or $1·08. Ans.

2. What is the interest of $1000 for 60 days, at 6 per cent per annum? Ans. $10·00.

3. What is the interest of $55·50 for 60 days, at 5 per cent.? 55·50×60÷72=46|25 or ·46¼ cts. Ans.

CASE IX.

To compute interest due on Notes, Bonds, &c. when partial payments are made.

Rule.—(1) Cast the interest up to the first payment; and if the payment exceed the interest, (2) deduct the excess

from the principal, and cast the interest upon the remainder to the time of the second payment. If the payment be less than the interest, (3) place it by itself, and cast on the interest to the time of the next payment, and so on until the payments exceed the interest, then deduct the excess from the principal and proceed as before.

Questions.—1. What is the first step in computing interest on Notes, Bonds, &c.?—2. What should you do, when the payment exceeds the interest?—3. What if the payment be less than the interest?

EXAMPLES.

Suppose A. should have a bond against B. for $1166 66 cents and 6 mills, dated May 1, 1796, upon which the following payments should be made, viz:

	Dolls. Mills.	*Months*	*Days.*
1. December 25, 1796 -	166,666	7	24
2. July 10, 1797 - -	16,666	6	15
3. September 1, 1798 -	50,000	13	21
4. June 14, 1799 - -	333,333	9	13
5. April 15, 1800 - -	620,000	10	1
What will be due upon it August 3, 1801?		15	18

Ans. $237,76.

TABLE OF OPERATION.—Set down the sum on which the interest is to be cast, with the time, interest and payments in columns thus.

	Principal.	Time.		Interest.	Payments.	Excess.
	Dolls Mills	*Mo.*	*Da.*	*Dolls. M.*	*Dolls. M.*	*Dolls. M.*
1	1166,666	7	24	45,499	166,666	121,167.
	121,167					
2	1045,499	6	15	33,978	16,666	
3	1045,499	13	21	71,616	50 000	
4	1045,499	9	13	49,312	333,333	
	245,093			154,906	399,999	245,093
5	800 406	10	1	40,153	620,000	579,847
	579,847					
	220,559	15	18	17,203		

The last remainder	220,559
Interest from the last payment	17,203
Sum due August 3, 1801	$237,762 Ans.

2. Supposing a note of 867 dollars 33 cents, dated Jan 6, 1794, upon which the following payments should be made, viz.

1. April 16, 1797	$136,44 cts.
2. April 16, 1799	319,
3. Jan. 1, 1800	518,68

What would be due July 11, 1801?

Ans. $215,103.

3. A note for $850, dated May 1, 1825 at 6 per cent., has the following endorsments:—

1 June 1, 1826	$150
2. Sept. 15, "	12
3. Jan. 1, 1827	215
4. July 16, "	10
5. Nov. 1, "	125
6. May 1, 1828	350

What remains due September 1, 1828?

Ans. $113,055.

A Second Method

Of computing interest on Notes, &c. where partial payments are made.

Rule.—(1) Find the amount of the principal for the whole time; then (2) compute the interest on the several payments, from the time they were paid to the time of settlement, and find their amount.

(3) Deduct the amount of the several payments from the amount of the principals the remainder will be the balance due.

Questions.—1. What is the 1st step in the 2d method of computing interest on notes &c.?—2. What the 2d step?—3. What the 3d and last step?

EXAMPLES.

1. A note for $850, dated May 1, 1825, at 6 per cent. interest, has the following endorsments:—

Received	June 1, 1826	150
"	Sept. 15, 1826	12
"	Jan. 1, 1827	215
"	July 16, 1827	10
"	Nov. 1, 1827	125
"	May 1, 1828	350

How much remains due September 1, 1828? Ans. $100 91·3*

2. Required the balance due on the 1st and 2d examples in method first.

Simple Interest in Lawful Money.

Rule.—(1) Multiply the principal by the rate per cent., and divide the product by 100, the quotient is 1 year's interest; if months and days are mentioned in the time, (2) find 1 year's interest as above, and divide it by the aliquot parts of a year, add the parts together, the sum will be the answer.

Questions.—1. What is the rule for casting interest in Lawful money? 2. How do you proceed when months and days are mentioned?

EXAMPLES.

1. What is the interest of £2 6 7½ for 1 year at 6 per cent.?

£.	s.	d.
2	6	7½
		6

	£.	s.	d.
100)13	19	9(0	2	9 57/100 Ans.
20			
100)279(2s.			
200			
79			
12			
100)957(9d.			
900			
57			

* Note.—By this last method the balance due in this example is $12·142 m. less than by the former method; and the reason for the difference is very obvious. By this 2d method, interest is supposed due only at the time of the last payment, whereas by the 1st method it is considered due annually, and that the interest of the principal for the first year is to be reckoned a part of the principal for the 2d year and so on, and that payments are made solely for the purpose of keeping down the interest. This I deem correct; for where a note is given without a specified time for payment, the interest becomes due yearly. The Courts of Law in Massachusetts have established the first of these methods for computing interest on Notes, Bonds, &c. where partial payments are made and is generally adopted by most of those who cast interest in all States in the Union.

When endorsements are made within the year the last *Method* may be used with propriety, but if made at or after the expiration of the year, I think it erroneous.

2. What is the interest of the same sum, viz. £2 6 7½ for one year, one month, ten days?

One year's interest, Fractions omitted.

1 month - - - -	$\frac{1}{12}$	2	9
10 days - - - - -	$\frac{1}{3}$	0	$2\frac{1}{4}$
		0	$0\frac{1}{12}$
		3	$0\frac{3}{12}$ Ans.

3. What is the interest of £250 15s. for 3 years, one month at 6 per cent.? Ans. £46 7 $9\frac{3}{4}$d.

COMPOUND INTEREST.

DEFINITION.—Compound Interest is the (1) interest of the interest; when interest is not paid yearly it ought to become (2) a part of the principal.

RULE.—(3) Find the interest of the principal for one year, and to the interest add the principal, this sum will be the principal for the second year; thus (4) continue to add the last interest and last principal together, until you have found the amount for the number of years required; (5) from the last amount subtract the first principal, and the remainder is the interest sought.

Questions.—1. What is Compound Interest?—2. When interest is not paid yearly what does it become?—3. What is the 1st step in the Rule?—4. What is the second step?—5. What is the last step?

EXAMPLES.

1. What is the compound interest upon 100 dollars for 3 years, at 6 per cent. per annum?

100	1st principal.
·06	
6·00	1st interest.
100·	1st principal.
106·00	2d principal.
·06	
6·3600	2d interest.
106·	2d principal added.
112·3600	3d principal.
·06	
6·741600	3d interest.
112·3600	3d principal added.
119 101600	3d amount.
100·000000	1st principal subtracted.
Ans. $19·1010	compound interest.

2. What is the compound interest of $125 for 2 years and 6 months, at 6 per cent.? Ans. $19·663.

METHOD SECOND.

RULE.—(1) Find what aliquot part the rate per cent. is of 100; (2) divide the principal by the aliquot part, or parts, and set the quotient, or quotients, under the principal, the same as quotients in short division;* (3) add the quotients and principal together, the sum is the second principal; (4) divide the second principal in the same manner, by the aliquot parts, &c. the sum of the quotients, &c. is the second principal; thus continue till you have found the amount required; (5) from the amount subtract the first principal, the remainder is the interest sought.

Questions.—1. What is the 1st step in method 2d, Compound Interest?—2. What is the 2d step?—3. What is the 3d step?—4. How then do you proceed?—5. What is the last step?

TABLES OF ALIQUOT PARTS OF AN HUNDRED.

1 per cent. is $\frac{1}{100}$	5 per cent. is $\frac{1}{20}$
2 per cent. is $\frac{1}{50}$	10 per cent. is $\frac{1}{10}$
4 per cent. is $\frac{1}{25}$	20 per cent. is $\frac{1}{5}$

EXAMPLES.

1. What will 112 dollars amount to in 4 years, at 6 per cent. compound interest?

5 per cent. is equal to $\frac{1}{20}$	112·00	
1 per cent. is equal to $\frac{1}{5}$ of 5 $\frac{1}{5}$	5·60 1·12	1 year's int.
5 per cent. is equal to $\frac{1}{20}$	118·72	1st amount.
1 per cent. is equal to $\frac{1}{5}$ of 5 $\frac{1}{5}$	5·936 1.187†	2d yrs. int
5 per cent. is equal to $\frac{1}{20}$	125·843	2d amount.
1 per cent. is equal to $\frac{1}{5}$ of 5 $\frac{1}{5}$	6·292 1·258†	3d yrs. int.
5 per cent. is equal to $\frac{1}{20}$	133·393	3d amount.
1 per cent. is equal to $\frac{1}{5}$ of 5 $\frac{1}{5}$	6·669† 1·333†	4th yrs. in.
	Amount $141·395	Ans.

*If the rate per cent. is no aliquot part of an hundred, use two, or more numbers that will make the rate per cent. and are aliquot parts of an hundred: divide by these numbers, add the quotients, the sum will be the answer.

2. What is the amount of $2·12 cents for 4 years, at 12 per cent. per annum? Ans. $3·33†.

3. What is the compound interest of $33·33 cents 2 yrs. at 10 per cent.? Ans. $6·99·9†.

4. What is the amount of 6·66 cents 2 years, at 9 per cent. per annum? Ans. $7·91·2†.

Method Third.

The third and easiest method is by the help of the following table, which shews the amount of one dollar for any number of years under 30, at 6 per cent. compound interest.

yrs.	*amt.*	*yrs.*	*amt.*	*yrs.*	*amt.*
1	$1·0600	11	$1·89829	21	$3·39956
2	1·12360	12	2·01219	22	3·60353
3	1·19101	13	2·13292	23	3·81975
4	1·26247	14	2·26090	24	4·04899
5	1·33822	15	2·39655	25	4·29187
6	1·41852	16	2·54035	26	4·54938
7	1·50363	17	2·69277	27	4·82234
8	1·59384	18	2·85434	28	5·11168
9	1·68948	19	3·02559	29	5·41838
10	1·79084	20	3·20713	30	5·74349

Use of the preceding table.

In the table find the amount of one dollar for the given time, and multiply the given principal by the amount of one dollar, the product is the answer.

EXAMPLES.

1. What will 50 dollars amount to in 6 years, at 6 per cent. compound interest?

$1 in 6 yrs. amounts to $1·41852×50=$70·926 Ans.

2. What will 505 dollars amount to in 5 years, at 6 per cent. compound interest? $675·801† Ans.

3. What will $222·22 amount to in four years, at 6 per cent. compound interest? Ans. 280·546†.

COMMISSION, BROKERAGE AND INSURANCE.

Definition.—*Commission* is (1) an allowance of so much per cent. to an agent for transacting business for another.

Brokerage is (2) an allowance of so much per cent. to a

person called a broker, for assisting merchants or factors in purchasing or selling goods.

Insurance is (3) a contract by which certain persons, or companies, agree to make good, losses of property by fire, storms, &c. in consideration of the payment to the insurer of so much per cent. on the value of the property insured.

Premium is (4) the sum paid by the owner of the property for the insurance.

Policy is (5) the written contract of insurance.

The Policy should always cover (6) a sum equal to the estimated value of the property insured, together with the premium; that is, a policy to secure the payment of 100 dollars at 2 per cent. must be out for 102 dollars.

RULE.—(7) Multiply the sum on commission, brokerage or insurance, by the rate per cent. and the product will be the commission, allowance or premium on the same.

Questions.—1. What is *Commission?*—2. What is *Brokerage?*—3. What is *Insurance?*—4. What is meant by *Premium?*—5. What is to be understood by *Policy?*—6. What amount should the policy always cover? 7. What rule have we for their operation?

EXAMPLES.

1. At 3 per cent. commission, how much must I allow for selling 525 dollars worth of goods? Ans. $15·75.

2. What is the allowance, should I transact business, to the amount of 827 dollars 67 cents, at 2½ per cent?
Ans. $20·69·1 m

3. If a broker sell goods to the amount of $2364·46, what is his demand at 1½ per cent.? Ans. $42 96·6 $\frac{3}{10}$ m.

4. What is the premium on 2525 dolls. at 10 per cent?
Ans. $252·50

5. What sum must a policy be taken out for to cover $2525, when the premium is 10 per cent.?

	$		$	$ cts. m	
Policy	100				
Premium	10				
Sum cov'd	90	: 100 :	:	2525=2805·55	5⅝ Ans.

Proof.

$ cts. m.	
2805·55·5⅝ at 10 per cent.	=$280·55·5⅝ Prem.
280·55·5⅝	
$2525 sum covered.	

6. What sum must a policy be taken out for, to cover 1800 dollars when the premium is 10 per cent.? Ans. $2000

DISCOUNT.

DEFINITION.—Discount is (1) an allowance made for the payment of any sum of money before it becomes due, and is the difference between that sum, due sometime hence, and its present worth.

The *present worth* of any sum or debt due some time hence, is (2) such a sum as, if put to interest, would, in that time and at the rate per cent for which the discount is to be made, amount to the sum or debt then due.

CASE I.

RULE.—(3) As the amount of 100 dollars for the time given is to 100 dollars; so is the given sum to its present worth: subtract the present worth from the given sum, the remainder is the discount.

Questions.—1. What is *Discount?*—2. What is the *present worth* of any sum due some time hence?—3. Give the rule for the operation?

EXAMPLES.

1. A. owes B. 1000 dollars payable in one year; A. offers to pay the ready money if B. will make discount at 6 per cent.; I demand the discount to be made.

dols. *dols.* *dols.* *dols.*
As 106 : 100 : : 1000 : 943·39$\frac{6}{106}$ present worth.

dols. *dols.* *dols.*
And 1000—943·39$\frac{6}{106}$=56·60 $\frac{4}{106}$ discount Ans.*

2. What discount must be made on 560 dollars due in 9 months; discount at 8 per cent. per annum?

Ans. $31·69$\frac{36}{106}$ cts.

*NOTE.—It is thought by many, that the interest of the sum for the time is the discount which ought to be made; but this is an error. I shall here observe if B. had discounted the interest of 1000 dollars for a year, the discount would have been 60 dollars; and he would have had but 940 dollars in ready money, which allow him to receive, and put at interest to a third person; and at the end of a year he would receive but $996·40 cts. for the 1000 dollars; therefore he receives $3·60 less, than he would have received of A. had it remained in his hands; but allow B. to receive $943·39$\frac{36}{106}$ cents of A. in ready money; and put it at interest to D. in one year it would amount to just 1000 dollars; Therefore by this rule neither party suffers any injury.

EXAMPLES.

3. What is the discount upon 1000 dollars due in a year; at 6 per cent. per annum?

Ans. $56·60$\frac{4}{1\,8\,8}$.

4. What is the discount upon 560 dollars, due 9 months hence; at 8 per cent. per annum?

Ans. $31·69$\frac{8}{1\,8\,8}$.

5. What is the discount upon 75 dollars due in 3 years; at 3 per cent. per annum?

Ans. 6·19$\frac{2\,0}{1\,8\,9}$.

EQUATION OF PAYMENTS

DEFINITION.—The use of this rule is (1) to find a certain time to make a payment of several sums of money at once, due at several different times.

RULE.—(2) Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the equated time.

Questions.—1. What is the use of the rule of *Equation of Payments*? —2. What is the rule for the operation?

EXAMPLES.

1. A. owes B. 100 dollars payable in 9 months; 72 dollars payable in 6 months; and 200 dollars payable in 22 months: in what time may A. pay all the notes at once, that neither party shall not be liable to suffer any harm?

First payment	100×	9 months	= 900
second do.	72×	6 months	= 432
third do.	200×	22 months	=4400

372 divisor. 5732 dividend.

5732÷372=15 mo. 12$\frac{2\,6}{3\,7\,2}$ days Ans.

2. C. owes D. 100 dollars on demand; 100 dollars in one year; 100 dollars in two years; 100 dollars in three years; what is the equated time to pay all at once? Ans. 1½ yr.

3. I owe $65·125, to be paid ¼ in 3 months, ½ in 5 months, ⅛ in 10 months, and the remainder in 14 months, in what time ought the whole to be paid? Ans. 6¼ months.

4. C. owes D. $1400, to be paid in 3 months; but D. being in want of money, C. pays him at the expiration of 2 months $1000; how much longer than 3 months ought C. in equity, to defer the payment of the rest? Ans. 2½ mo.

BARTER.

BARTER is (1) the exchanging of one commodity for another, and directs merchants so to proportion their goods, that neither may sustain loss.

RULE.—(2) Find the value of the commodity whose quantity is given; then find what quantity of the other, at the proposed rate, can be bought for the same money.

Questions.—1. What is Barter?—2. What is the general rule for its operation?

EXAMPLES.

1. How many reams of paper, at 3 dollars and 50 cents per ream must be given in barter for 50 yards cloth, at 5 dollars and 25 cents per yard? Ans. 75 reams.

$ cts. yds. $ cts.
5·25×50=262·50 value of the cloth.
$ cts. R'm. $ cts. Reams.
3·50 : 1 : : 562·50 : 75 Ans.

2. A. has rum at $1·50 per gallon, and B. has 4 pipes of wine, containing 125 gallons each, at $3·50 per gallon, how much of A's. rum can B have for his wine?
Ans. 1166⅔ gallons.

3. A farmer sold in market 80 bushels of rye, at 62 cts. per bushel; 100 bushels corn at 65 cents; 50 bushels oats at 30 cents; for which he received $40 cash; 60 bushels salt, at 60 cents per bushel; 10 quintals of fish at $2·50 cents; 1 cwt. sugar at $10·40 cents; 5 gallons brandy at $1·25 cents; and the balance in rice at 4 cents per pound. How many pounds of rice did he receive? Ans. 298¾ lbs.

4. A. has B's. note for $450, with 8 months interest due on it; and to redeem it, B. gives him 200 bushels oats at 30 cents per bushel; 150 bushels salt at $1·25 per bushel, and the balance in flour at 4 cents per pound; how many pounds of flour does A. receive? Ans. 551¼ lbs.

5. A. has cambrick worth $10 per yard, but, in bartering, he will receive $15; B. has linen cloth, worth $2·30 per yard ready money: at what price ought B. to rate his linen to be in proportion with A's. bartering price?
Ans. $3·45 cents.

6. A. has linen cloth at 30 cts. per yard, ready money, in barter 36 cts.; B. has 3610 yards of ribband at 22 cts. per yard ready money, and would have of A. 200 dollars in

ready money, and the rest in linen cloth; what rate does the ribband bear in barter per yard, and how much linen must A. give B.? Ans. ·26·4 m. rate of ribband.
1980¾ yds. linen.

LOSS AND GAIN.

DEFINITION.—*Loss and Gain* is (1) a rule which discovers what is gained or lost in buying and selling goods; and instructs merchants and dealers (2) to raise or fall their price so as to gain or loose so much per cent.

This rule is (3) founded on proportion, and the questions belonging to it are performed by the *Rule of Three.*

RULE. I.—*If it be required to know what is gained or lost per cent.;* first find what the gain or loss is by *Subtraction.*

Then (4) say by the *Rule of Three Direct,* As the price it cost, is to the gain or loss; so is 100, to the gain or loss per cent,

II. *If to know how an article must be sold to gain or lose so much per cent.;* (5) say,

As 100, is to the price; so is 100, with the gain added or loss subtracted, to the gain or loosing price.

III. *If to know what an article cost, when there is gained or lost so much per cent.;* (6) say,

As 100 with the gain added or loss subtracted, is to the price; so is 100, to the prime cost.

IV. *If to know what would be gained or lost per cent., if goods, sold at such a rate, there is gained or lost so much per cent. if sold at another rate;* (7) say,

As the first price, is to 100, with the gain added or loss subtracted; so is the other price, to the gain or loss per cent at the other rate.*

Questions.—1. What is *Loss and Gain*?—2. What does it teach merchants?—3. On what is this rule founded, and by what are the questions performed?—4. What is the rule when it is required to know what is gained or lost per cent.?—5. What is the rule when it is required to know how an article must be sold to gain or lose so much per cent.?—6. What is the rule, when it is required to know what an article cost, when there is gained or lost so much per cent.?—7. How do you find what would be gained or lost per cent. if goods sold at such a rate, there is gained or lost so much per cent. if sold at another rate?

* NOTE.—If the answer is 100, there is neither gain nor loss; the excess of 100, is gain; the deficiency is loss per cent.

EXAMPLES.

1. If I purchase a horse for $60 and sell him again for $90; what do I gain per cent.? Ans. 50 per cent.

$60 : $90 − 60 = $30 : : $100 : 50 per cent Ans.

2. If I buy salt for 84 cents a bushel, and sell it for 1·12 a bushel, what do I gain per cent.? Ans. $·33\frac{1}{3}$ per cent.

3. Bought 18 cwt. of cheese at 28s. per cwt. which I sell out again for $3\frac{1}{2}$d. per lb. what is the whole gain, and gain per cent.? Ans. £4. 4s. whole gain.
$16\frac{2}{3}$ gain per cent.

4. If goods worth $900 are shipped and come to a bad market; what will be the amount, if sold at 15 per cent loss? Ans. $765.

5. How must I sell tea per lb. that cost me 13s. 5d. to gain after the rate of 25 per cent.? Ans. 16s. $9\frac{1}{2}$d.

6. When 40 per cent. loss is made on coffee, sold at 12 cents per lb.; what was the first cost? Ans. 20 cents.

7. I sold a horse for 60 dollars, and by so doing lost 20 per cent. whereas I ought in trading to have cleared 30 per cent.; what was the real value of the horse?
Ans. $97 50.

8. If wine, sold at $1·50 cents per gallon, be 15 per cent, profit; what would the gain or loss have been per cent. had it been sold at $1·25 cents per gallon?
Ans. $4·16\frac{2}{3}$ loss per cent.

9. Bought 115 gallons of rum at $1 per gallon; how many gallons of water must be put in so as to gain 5 dollars by selling it at the same price it was bought?
Ans. $16\frac{1}{2}$ gallons.

SINGLE FELLOWSHIP.

DEFINITION.—Single Fellowship teaches (1) to find the gain or loss of each, of several persons, who have joined their stocks for an equal time in trade.

RULE.—(2) Say by proportion as the sum of all their stocks is to the sum of gain or loss, so is each man's particular stock, to his particular share of the gain or loss.

PROOF.—(°) Add the several shares of gain or loss together, if the work is right, the sum will be equal to the sum of gain, or loss.

Questions.—1. What does *Single Fellowship* teach?—2. What is the general rule?—3. What is the method of proof?

EXAMPLES.

1. A. B. C. and D. joined their stocks, and bought 367 barrels of beef which they sold at $2·75 cts. advance upon the barrel; A.'s stock was 562 dollars; B.'s 646 dollars; C.'s 394 dollars; and D.'s 600 dollars: I demand the share of each?

367 bar. at $2·75 cents=$1009·25 cents gained.
562 A.'s stock.
646 B.'s do.
394 C.'s do.
600 D.'s do.

	dols. cts.		dols.	dols. cts.	
As 2202	: 1009·25	: : 562	:	257·58 $\frac{7}{2}\frac{3}{2}\frac{4}{0}\frac{2}{2}$	A.'s share
2202	: 1009·25	: : 646	:	296·08 $\frac{7}{2}\frac{3}{2}\frac{4}{0}\frac{2}{2}$	B.'s do.
2202	: 1009·25	: : 394	:	180·58 $\frac{7}{2}\frac{3}{2}\frac{4}{0}\frac{2}{2}$	C.'s do.
2202	: 1009·25	: : 600	:	275·00	D.'s do.

1009·25 proof.

2. Three men built a ship which cost 4560 dollars; $\frac{1}{4}$ of her belonged to A.: $\frac{2}{5}$ to B.: and the remainder to C.: they agreed to freight her with a cargo of lumber worth 1270 dollars, in proportion to their shares: and send her to the W. I. She was lost; I demand the loss of each.

Ans. A. lost $1457·5: B. lost $2332: C. lost $2040·5.

3. Divide the number 360 into 4 such parts, which shall be to each other as 3, 4, 5, and 6.

60
80
100
120 } Ans.

360 *Proof.*

4. A. and B. companied; A. put in $150 and took $\frac{3}{5}$ of the gain; what did B. put in? Ans. $100

DOUBLE FELLOWSHIP.

DEFINITION.—Double Fellowship teaches (1) to find the shares of gain, or loss of several persons, who have joined their stocks in trade for unequal times.

RULE.—(2) Multiply each man's stock by the time it was in trade; then (3) say, as the sum of the products is to the gain or loss, so is the product of each man's stock and time, to his share of the gain or loss.

The method of proof is (4) the same as in Single Fellowship.

Questions.—1. What does Double Fellowship teach?—2. What is the 1st step in the operation?—3. What is the 2d and last step?—4. What is the method of proof?

EXAMPLES.

1. A. commenced trade with a capital of 500 dollars; in 1 month after, he received B. as a partner, with a capital of 450 dollars; in 3 month's after this, they received C. with a capital of 900 dollars; they traded 4 years from the time that A. commenced business, and gained 3420 dollars; I demand the gain of each?

stock mo.

A's stock multiplied by the time $500 \times 48 = 24000$ A.'s
B's stock multiplied by the time $450 \times 47 = 21150$ B.'s
C's stock multiplied by the time $900 \times 44 = 39600$ C.'s

total amount 84750

amt.	*dols.*		*dol's. cts.*	
84750 :	3420 : :	24000 :	$968 \cdot 49\frac{17}{47}$	A.'s share.
84750 :	3420 : :	21150 :	$853 \cdot 48\frac{57}{94}$	B.'s do.
84750 :	3420 : :	39600 :	$1598 \cdot 01\frac{65}{94}$	C.'s do.

2. A. began trade with a capital of 1300 dollars; he traded 9 months and died: B. offered the family to continue the business, if they would allow his time to be equal to A.'s stock; and divide the gain accordingly; it was so agreed, and he continued business 1 year and three months from the death of A; I demand the shares of each, allowing the gain to have been 392 dollars.

A.'s capital 1300 dollars, and in trade 2 years. B.'s time valued 1300 dollars, and in trade, 1 year and 3 months.

The heirs of A. had $\$241 \cdot 23\frac{39}{107}$
B. $\$150 \cdot 76\frac{68}{107}$ Ans.

3. B. and D. joined stocks, viz. 500 dollars each; they traded 2 years, and B. took out one fifth of his stock, and they continued their trade 3 years longer, and gained 627 dollars; what is the share of each?

Ans. B.'s $293·48$\frac{4}{7}$; D.'s $333·51$\frac{3}{7}$.

4. A. B. and C. trade together; A. at first put in 480 dollars for 8 months, then put in 200 dollars more and continued the whole in trade 8 months longer, at the end of which he took out his whole stock; B. put in 800 dollars for 9 months, then took out $583,333 and continued the rest in trade 3 months; C. put in $366,666 for ten months, then put in 250 dollars more, and continued the whole in trade 6 months longer. At the end of their partnership they had cleared 1000 dollars; what is each man's share of the gain?

Ans. $378,827 A.'s share.
320,452 B.'s share.
300,721 C.'s share.

PRACTICE.*

DEFINITION.—Practice is a rule which teaches (1) to solve such questions by division, as are solved by compound multiplication or single rule of three.

Table of aliquot parts of a dollar.

1 cent	is = $\frac{1}{100}$ dol.	10 cents	is = $\frac{1}{10}$ dol.
2 "	is = $\frac{1}{50}$	20 "	is = $\frac{1}{5}$
4 "	is = $\frac{1}{25}$	50 "	is = $\frac{1}{2}$
5 "	is = $\frac{1}{20}$	25 "	is = $\frac{1}{4}$

Table of aliquot parts of a Cwt. & qr.

1 qr. is = $\frac{1}{4}$ cwt.	7 lb. is = $\frac{1}{4}$ qr.
2 " is = $\frac{1}{2}$ cwt.	14 " is = $\frac{1}{2}$ "

CASE I.

When the price is cents and the quantity whole numbers

RULE.—(2) Write down the quantity, as so many dollars,

* NOTE.—The rule of Practice is not so useful in the present currency of the United States as it was in the former currency; but in many instances it is still a short and easy way of finding the price of several things.

see what part, the cents in the price, is of a dollar, divide by that part, and the quotient is the answer.

Questions.—1. What does the rule of Practice teach?—2. What is the rule for finding the amount when the price is cents and the quantity whole numbers?

EXAMPLES.

1. What will 622 yards of India cotton come to, at 25 cts. per yard? * $622÷¼=$155·50 Ans.

2. What is the price of 3472 oranges at 5 cents a piece?
Ans. $173·60 cts.

CASE II.

When the quantity is whole numbers and the price is dollars and cents.

RULE.—(1) Multiply the quantity by the dollars in the price, and work for the cents as in case first.

Question.—1. What is the method of operation when the quantity is whole numbers and price dollars and cents?

EXAMPLES.

1. What will 22 thousand of boards come to, at 9 dols. 25 cents per thousand?

25 cts. =¼)22 quantity.
9

198
5·50

$203·50 Ans.

2. What will 20 barrels of cider come to at 2 dollars 16 cents per barrel? Ans. $43·20 cts.

CASE. III.

When the price is no aliquot part of a dollar.

RULE.—(1) Divide by two or more numbers, whose sum will make the number required.

Question.—1. When the price is no aliquot part of a dollar, what is your method of operation?

* —The quantity being written down as so many dols. I considered that 25 cts. were equal to ¼ of a dollar, therefore ¼ of the quantity is the answer.

EXAMPLES.

1. What will 622 yards of linen come to, at 30 cts. per yard.

	dols.cts.
20 cts. $=\frac{1}{5}$	)622·00
10 cts. $=\frac{1}{2}$ of 20. $\frac{1}{2}$	)124·40 cts. price at 20 cts.
	62·20 price at 10 cts.
Ans.	$186·60 price at 30 cts.

2. What will 642 yards of India cotton come to, at 17 cents per yard?

	dols.	
2 cts. $=\frac{1}{50}$	} 642·00	
10 " $=\frac{1}{10}$	}	
	12·84	price at 2 cts.
" " $=\frac{1}{2}$ of 10. $\frac{1}{2}$	)64·20	price at 10 cts.
	32·10	price at 5 cts.
Ans.	$109·14	price at 17 cts.

CASE IV

When there are several denominations in the quantity, and the price is dollars and cents.

RULE.—(1) Multiply the dollars in the price into the whole numbers of the quantity; work for the cents in the price as in the preceding cases; and for the parts in the quantity divide by the aliquot parts of the price of one whole number, add the quotients together, the sum is the answer.

Question.—1. When there are several denominations in the quantity and the price is dollars and cents, what is the rule for the operation?

EXAMPLES.

1. What will 112 cwt. 3 qr. 14 lbs. of sugar come to, at 12 dols. 25 cts. per cwt.?

25 cts.=¼)112 cwt.
12 dols.

1344 price at 12 dols.
28 price at 25 cts.

$1372 price of 112 cwt. at $12·25.

For the parts in the quantity.

dols.cts.

3 qrs.=¾)12·25

14 lbs.=½ of 3 qrs. ½)9·18† price of 3 qrs.
1·53 price of 14 lbs.

10·71 price of 3 qrs. 14 lbs.
1372·00 price of 112 cwt.

$1382·71† Answer.

TARE AND TRET.

DEFINITION.—Tare and Tret are (1) allowances made the buyer by the seller, for the weight of the box, cask, &c.

Tare is (2) an allowance for the weight of the cask, and is generally at a certain rate per cwt. or per barrel, &c.

Tret is (3) an allowance for the waste by dust, and is generally at the rate of 4 lb. per every 104 lb. &c.

Cloff is (4) an allowance made the buyer for the turn of the scales, and is generally at 2 lb. per 3 cwt.

Gross is (5) the whole weight of the goods, together with the cask, or box, &c. that contains them.

Suttle is (6) when part of the allowance is deducted.

Neat is (7) the remainder after all deductions are made.

Questions.—1. What are *Tare and Tret?*—2. What is *Tare?*—3. What is *Tret?*—4. What is *Cloff?*—5. What is *Gross?*—6. What is *Suttle?*—7. What is *Neat?*

CASE I.

When the tare is at a certain rate per hhd. &c.

RULE.—(1) Multiply the tare per box, or hhd. by the number of boxes, &c. subtract the product from the gross weight, the remainder is the neat weight required.

Question.—1. When the *Tare* is at a certain rate per hhd. &c. how do you find the *neat* weight?

EXAMPLES.

1. What is the neat weight of 3 hhd. of tobacco, weighing in all 13 cwt. 2 qr. 27 lbs; tare at 1 cwt. 1 qr. per hhd.?

cwt.	*qr.*	*lb.*	*cwt.*	*qr.*	*cwt.*	*qr.*	*lb.*
13	2	27—1	1×3=9		3	27	Ans.

2. What is the neat weight of 910 bags of India sugar, each weighing 1 cwt. 2 qr. 20 lb.; tare at the rate of 15 lb. per bag? Ans. 1405 cwt. 2 qr. 14 lb.

CASE II.

When the tare is at a certain rate per cwt.

RULE.—(1) If the rate per cwt. is an aliquot part, divide the gross by the aliquot part, or parts, and the quotient, or quotients will be the tare of the whole, which must be subtracted from the gross; the remainder will be the neat weight required.

Question.—1. When *Tare* is at a certain rate per cwt. how do you find the neat weight?

EXAMPLES.

1. What is the neat weight of 19 cwt. 3 qr. 26 lb. gross tare at 2 qr. per cwt.?

	cwt.	*qr.*	*lb.*	
2 qr. = ½ of 1 cwt.	½)19	3	26	gross.
	9	3	27	tare subtract.
	Ans. 9	3	27	neat.

2. What is the neat weight of 347 bags of sugar, each weighing 2 cwt. 3 qr. 12 lb.; tare at 21 lb. per bag?
Ans 926 cwt. 1 qr. 13 lb.

CASE. III.

When the tare is at a certain rate per cwt. and is no aliquot part.

RULE.—(1) Say by single proportion as 112 lb. is to the rate per cwt. so is the number of lbs. gross to the tare required: which being subtracted from the gross, the remainder is neat.

Question.—1. When *Tare* is at a certain rate per cwt. and is no aliquot part, what *rule* do you have for the operation?

EXAMPLES.

1. What is the neat weight of 38 cwt. 0 qr. 4 lb. of tobacco; tare at 11 lb. per 112 lb.?

cwt. *qr.* *lb.*
38 0 4=4260 pounds.

lb. *lb.* *lb.*
As 112 : 11 : : 4260 : $418\frac{4}{112}$ tare.

lb. *lb.* *lb.* *cwt.* *qr.* *lb.*
4260—$418\frac{4}{112}$=$3841\frac{68}{112}$ neat=34 1 $5\frac{68}{112}$ Ans.

CASE IV.

When tare and tret are allowed.

RULE.—(1) Find and subtract the tare, as in the preceeding cases; the remainder is suttle; (2) divide the suttle by 26 (N. B. $\frac{4}{104}=\frac{1}{26}$) The quotient is the tret, which (3) subtract, and the remainder is the neat weight.

Questions.—1. When tare and tret are both allowed what is the 1st step in the operation?—2. What is the 2d step?—3. What is the 3d?

EXAMPLES.

1. What is the neat weight of 5 cwt. 2 qr. of allum; tare 12 lb. per cwt.; and tret 4 pounds per every 104?

cwt. *qrs.* *lb.* *lb.* *lb.* *lb.* *lb.*
5 2=616 As 112 : 12 : : 616 : 66 tare.

lb. *lb.* *lb.* *lb.* *lb.* *lb.*
and 616—66=550 suttle. 550÷26=$21\frac{4}{26}$ tret

lb. *lb.* *lb.* *cwt.* *qr.* *lb.*
and 550—$21\frac{4}{26}$=$528\frac{2}{26}$ neat.=4 2 $24\frac{2}{26}$ Ans.

2. What is the neat weight of 21 cwt. 1 qr. 26 lb. of rice tare 15 lb. per cwt. and tret 4 lb. per 104.

Ans. 17 cwt. 3 qr. $15\frac{8}{26}\frac{14}{104}$ lb.

CASE V.

When tare, tret, and cloff are allowed.

RULE.—(1) Find and subtract the tare and tret as in the preceding cases, and divide the remainder by 168, the quotient is cloff, which subtract, and the remainder is the neat weight sought.

Question.—1. What is the rule for finding the neat, when *tare*, tret and *cloff* are allowed?

EXAMPLES.

1. What is the neat weight of 10 cwt.; tare 14 lb. per cwt.; tret 4 lb. per 104; and cloff 2 lb. per every 3 cwt.?

10 Cwt. = 1120 lbs.

$\frac{14}{112} = \frac{1}{8}$	1120 140 tare sub.
$\frac{4}{104} = \frac{1}{26}$	980 37† tret sub.
$\frac{2}{3 \cdot 0} = \frac{1}{150}$	943 5† cloff sub.
	938 lbs. neat. = 8 cwt. 1 qr. 14 lb.

[Ans

2. What is the neat weight of 12 cwt. 2 qr.; tare 7 lb per cwt.; tret 4 lb. per 104; cloff 2 lb. per 3 cwt.?

Ans. 11 cwt. 0 qr. 24† lb.

ALLIGATION.

DEFINITION.—Alligation is (1) the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality. It is (2) of two kinds, *Medial* and *Alternate*.

ALLIGATION MEDIAL.

Alligation Medial is a rule made use of (3) to find the price of a compound of several things mixed together; the prices of the ingredients and quantities being known.

RULE.—(4) As the sum of all the quantities, is to the total value of all the quantities, so is any part of the mixture to the price of that part.

Questions.—1. What is Alligation?—2. Of how many kinds is Alligation, and what are they?—3. For what is *Alligation Medial* used?—4. What is the *rule?*

EXAMPLES.

1. A farmer mixes 4 bu. of blighted corn at ·42 cents per bu.; 6 bu. of oats at ·33 cents; 5 bu. of barley at ·96 cents; and 15 bu. of potatoes at ·20 cents per bu.; what is one bu. of the mixture worth?

	bu.		*cts. D.cts.*
Corn	4	at	·42=1·68
Oats	6	"	·33=1·98
Barley	5	"	·96=4·80
Potatoes	15	"	·20=3·00

As 30 : 11·46 : : 1 : 38⅟₂ Ans.

2. A trader mixes 60 gallons of rum at ·90 cts. with 62 gal. at ·80 cts.; 40 gal. at ·45 cts.; and 10 gal. of water: what is 10 gal. worth? Ans. $7·06† cts.

3. A trader mixes 1 cwt. of sugar at 10 dollars per cwt. with 3 cwt. at 12 dollars, and adds 40 lbs. of sand to the mixture; what is the value of 1 cwt. of the mixture?

Ans. $10·55†.

Alligation Alternate.

Definition.—Alligation Alternate is a rule made use of (1) to find the quantity of any number of simples, whose prices are given, that will make a mixture worth a certain price, therefore it is the reverse of alligation medial, and may be proved thereby.

CASE I.

Rule.—(2) Write down the prices of the several simples in a column, one above the other, beginning with the least, &c. (3) on the left of the column draw a perpendicular line, and on the left of the line, write down the price of the mixed quantity; then (4) connect with a curved line each of the simples which is less, with one that is greater than the mixed quantity; and (5) set the difference between the least simple (and price of the mixed quantity) against the simple that is largest; and the difference of the larger against that which is less, &c. After having connected each simple that is less with one that is greater, and found the differences, the figures standing against each simple will express (6) the quantity that must be taken of that price, to form a mixture of the price intended.

Questions.—1. For what is Alligation Alternate used?—2. What is the 1st step in the *rule* Case 1st?—3. What the 2d step?—4. What the 3d step? 5. What the 4th?—6. After finding the differences, what will those figures standing against each simple express?

EXAMPLES.

1. A farmer would make a mixture of corn at ·50 cents; oats at ·25 cents; barley at ·90 cents; and potatoes at ·20 cents per bushel: I demand the quantity of each that must be taken that the mixture may be worth ·30 cents per bushel.

The price of the Mixture cts. ·30

20	60 bushels of potatoes,	will make
25	20 bushels of oats,	a mixture
50	5 bushels of corn,	worth ·30
90	10 bushels of barley,	cts. per bu.

Proof by aligation medial.

Bushels 60×·20 cts.=$12·00
20×·25 cts.= 5·00
5×·50 cts.= 2·50
10×·90 cts.= 9·00

				bush.		*cts.*
As 95	:	$28·50	: :	1	:	·30 Ans.

The first example linked another way.

The price of the mixture. cts. ·30

20	20 bushels of potatoes,	will make
25	60 bushels of oats,	a mixture
50	10 bushels of corn,	worth ·30 cts.
90	5 bushels of barley,	per bushel.

Proof by aligation medial.

Bushels 20×·20 cts.=$4·00
60×·25 cts.=15·00
10×·50 cts.= 5·00
5×·90 cts.= 4·50

				bu.		*cts.*
As 95	:	$28·50	: :	1	:	·30 Ans.

2. A seller of wines would mix wine at ·90 cts. ·30 cts. ·70 cts. ·60 cts. and ·50 cts. per gallon; and take such a quantity of each, that the mixture may be worth ·85 cts. per gallon; I demand the quantity of each that may be taken.

This question admits of a variety of linkings.

The price of the mixture cts. ·85.

			cts.
50	5		5 at ·50
60	5	Ans.	5 at ·60
70	5		5 at ·70
80	5		5 at ·80
90	35+25+15+5=		80 at ·90

NOTE.—Questions in this rule admit of a great variety of answers, according to the manner of their being linked together, and yet the value of the mixture will be the same.

CASE II.

When the mixture is limited to a certain quantity.

RULE.—(1) Find the quantity that will compose a mixture by case first: then (1) say *by proportion* as the sum of the quantities thus found, is to the limited quantity, so is each particular quantity found, to the part of that quantity.

Questions.—1. What is the first step in the operation, when the mixture is limited to a certain quantity?—2. What is the second and last step?

EXAMPLES.

1. A seller of liquors would make a cask of cherry of 80 gallons: and would compose it of rum at 50 cts. and ·90 cts. per gallon, and of water: I demand the quantity that must be taken, allowing the mixture to be worth ·60 cents per gallon.

The price of the Mixture cts. ·60.

00	30	30	gallons of water.
50	30	30	gallons of N. E. rum.
90	60+10=	70	gallons of W. I. rum.

	Galls.	galls.	galls.		
As	130	: 80	: : 30	: $18\frac{60}{130}$	of water.
	130	: 80	: : 30	: $18\frac{60}{130}$	of N. E. rum.
	130	: 80	: : 70	: $43\frac{60}{130}$	of W. I. rum.

Ans.

CASE III.

When one of the simples is limited to a certain quantity.

RULE.—1 Find the quantities that will make a mixture by case first (the simple that is limited must be linked) then 2 say, as the quantity that stands against the price of the limited quantity is to the other quantities taken separately, so is the limited quantity to the quantity of each.

Questions.—1. What is the first step in the operation, when one of the simples is limited to a certain quantity?—2. What is the 2d and last step?

EXAMPLES.

1. A grocer has 40 lbs. of tea worth ·80 cts. per lb. which he would mix with other teas, at ·90, ·80, and ·70 cts. per lb.; what quantity of each must be taken that the mixture may be worth ·60 cts. per lb.?

The price of the Mixture.				
cts. ·60.	30	30+20+10=60		
	70	30	Answers.	
	80	30		
	90	30		

lbs.	lbs.		lbs.	lbs.		lbs.	cts.	proof
As 60	: 30	: :	40	: 20	Answers.	20×	·70=	$14·00
60	: 30	: :	40	: 20		20×	·80=	16·00
60	: 30	: :	40	: 20		20×	·90=	18·00
The limited quantity				40		40×	·30=	12·00
						100 lbs.		$60·00

As 100 lb. : $60 : : 1 lb. : 60 cts. Ans.

POSITION.

Position is (1) a rule which by false or supposed numbers, taken at pleasure, discovers the true one required. It is (2) of two kinds, *Single* and *Double*.

SINGLE POSITION

Is (3) the working of one supposed number, as if it were the true one, to find the true number.

Rule.—(4) Take any number and perform the same operations with it as are described to be performed in the question.

Then (5) say as the sum of the errors is to the given sum, so is the supposed number to the true one required.

Proof.—(6) Add the several parts of the sum together, and if it agree with the sum, it is right.

Questions.—1. What is Position?—2. Of how many kinds is Position, and what are they?—3. What is *Single Position?*—4. What is the 1st step in the rule?—5. How then do you proceed?—6. What is the method of *Proof?*

EXAMPLES.

1. Two men, A. and B. having found a bag of money, disputed who should have it. A. said the half, third, and one fourth of the money made 130 dollars, and if B. could tell how much was in it, he should have it all, otherwise he should have nothing; I demand how much was in the bag?

Suppose 60 *dollars.* Ans. $120.

										Proof.
The half	30									120
— third	20									
— fourth	15									60=½
			$		$		$			40=⅓
	65	:	130	: :	60	:	120	Ans.		30=¼
										130=given sum.

2. A. B. and C. talking of their ages, B. said his age was once and a half the age of A.; and C. said his age was twice and one tenth of the age of both, and that the sum of their ages was 93; what was the age of each?

Ans. A.'s 12, B.'s 18, C.'s 63 years.

3. Seven eighths of a certain number exceeds four fifths of it by 6; what is that number? Ans. 80.

4. If $\frac{5}{16}$ of the years I have lived be multiplied by 7, and $\frac{1}{2}$ of them be added to the product, the sum being 292, what is my age? Ans. 60 years.

5. A schoolmaster, being asked how many scholars he had, said, if I had as many, half as many, and one quarter as many more, I should have 264; how many had he?

Ans. 96.

6. I lent a friend a sum of money unknown, at 6 per cent. simple interest; at the end of 4 years I received for the principal and interest $496; what sum did I lend?

Ans. $400

DOUBLE POSITION.

Double Position is (1) that which discovers the true number, or numbers sought, by making use of two supposed numbers.

Rule.—(2) Take any two numbers and proceed with them according to the conditions of the question.

(3) Place each *error* against its respective *position or supposed number;* if the *error* be too great, mark it with +; if too small, with — *

(4) Multiply them cross-wise, the first *positon* by the last *error*, and the last *position* by the first *error*.

(5) If they be alike, that is when both are greater, or both less than the given number, divide the difference of the *products* by the difference of the *errors*, and the quotient will be the answer; but if the *errors* be unlike, (6) divide the sum of the *products* by the sum of the *errors*, and the quotient will be the answer.

Questions.—1. What is *Double Position?*—2. What is the 1st step in the *rule?*—3. What is the 2d step?—4. What is the 3d step?—5. When are the *Errors* said to be *alike*, and if alike, how do you proceed?—6. If the *errors* be *unlike*, that is, one larger and the other less, how do you proceed?

* These two marks or chareters signify thus: $\left\{\begin{array}{l}+\text{plus or more.}\\ -\text{minus or less.}\end{array}\right.$

EXAMPLES.

1. A man lying at the point of death, left to his three sons all his estate, viz. to F. half wanting 50 dollars; to G. one third; and to H. the rest, which was 10 dollars less than the share of G. I demand the sum left, and each son's share. Ans. $360.

Operation.

Suppose the sum 300 dollars.

Then 300÷2—50=100 F.'s part.
300÷ 3=100 G.'s part.
G.'s part 100—10= 90 H.'s part.

Sum of all their parts 290

Again suppose the sum 900 dollars.

Then 900÷2—50=400 F.'s part.
900÷ 3=300 G.'s part.
G.'s part 300—10=290 H.'s part.

Sum of all their parts 990

Suppose.	Error 10— Errors
300	10—
900	90+
9000	27000
27000	

Error 90+

Having proceeded with the supposed numbers according to the conditions of the question, the *sum of all their parts* must be subtracted from the supposed number; thus the 290 is subtracted from 300, the supposed number, &c.

Dollars.

Sum of Errors. } 100)36000(360 Ans.

The divisor is the sum of the errors 90+ and 10-

2. There is a fish whose head is 10 feet long; his tail as long as his head and half the length of his body, and his body as long as his head and tail; what is the whole length of the fish? Ans. 80 feet.

3. A certain man having driven his swine to market, viz. hogs, sows and pigs, received for them all £50; being paid for every hog 18s. for every sow 16s. for every pig 2s.; there were as many hogs as sows, and for every sow there were three pigs; I demand how many there were of each sort? Ans. 25 hogs, 25 sows, and 75 pigs.

4. A laborer was hired for 40 days upon these conditions, that he should receive $2 for every day he wrought, and forfeit $1 for every day he was idle; at the expiration of the time he received $50; how many days did he work, and how many was he idle? Ans. wrought 30 days.
idle 10 "

5. A. and B. laid out equal sums of money in trade; A. gained a sum equal to ¼ of his stock, and B. lost 225 dollars; then A.'s money was double that of B.'s; what did each one lay out? Ans. $600.

6. A man had 2 silver cups of unequal weight, with one cover for both, weight 5 oz.; now if he put the cover on the less cup, it will be double the weight of the greater; and put it on the greater cup, it will be three times the weight of the less cup, what is the weight of each cup?

Ans. 3 oz. less cup, and 4 oz. greater cup.

7. A. and B. have the same income; A. saves ⅓ of his; but B. by spending 30 per annum more that A., at the end of 8 years finds himself 40 dollars in debt; what is their income, and what does each spend per annum?

Ans. Their income is $200 per annum. A. spends $175, and B. 205 per annum.

8. When first the marriage knot was tied betwixt my wife and me,
My age did hers as far exceed, as three times three does three;
But after ten, and half ten years, we man and wife had been,
Her age came up as near to mine, as eight is to sixteen. [day.
Now Tyre skill'd in numbers, say, what were our ages on the wedding
Ans. Sir, *forty five years* you had been,
Your bride no more than just fifteen.

EXTRACTION OF THE SQUARE ROOT.

DEFINITION.—Extracting the square root, is (1) the finding of a number, which being multiplied into itself will produce the given number, or is finding the root of a square number.*

Roots	1,	2,	3,	4,	5,	6,	7,	8,	9.
Squares	1,	4,	9,	16,	25,	36,	49,	64,	81.

RULE —(2) Point the given number into periods of two figures each; beginning at the place of units, thus 134567: If the point happen to fall upon the last figure, it must be considered as (3) a full period, thus 12345; secondly, having pointed the numbers into periods of two figures each; (4) begin at the left hand, and find the greatest square that can be had in that period; place the root thereof in the quotient, and its square under the first period, subtract it therefrom, and to the remainder bring down the next period, and call it the resolvend; (5) double the quotient or root, and use it for a divisor; divide the resolvend, omitting the right hand

* A number is squared when it is multiplied into itself.

figure, and place the answer in the quotient, and also at the right of the last divisor; (6) multiply the divisor by the figure last put on its right, (and in the quotient;) place the product under the resolvend, and subtract it therefrom, and to the remainder, bring down the next period; (7) double the right hand figure of the last divisor, and use it for a new one; divide the resolvend as before, omitting its right hand figure; thus continue, until the periods are all brought down, the quotient is the root sought.

Questions.—1. What is it to *extract the square root?*—2. What is the 1st step in the operation?—3. How is the last figure considered, when the point falls upon it?—4. What is the 2d step?—5. What the 3d?—6 What the 4th?—7. What the 5th, and last step?

EXAMPLES.

1. The square root of 37491129 is required.

6×6=36 greatest sqr.)37491129(6123 root or Answer.
36

121)149
121

1222)2811
2444

12243)36729
36729

2. What is the square root of 8896451041? Ans. 94321.
3. I demand the square root of 10201. Ans. 101.
4. The square root of 36481 is required. Ans. 191.

NOTE.

If a remainder is left after the periods are all brought down.

RULE.—(1) Annex periods of cyphers, and continue the operation to any exactness; the root thus found must be expressed (2) decimally.

Questions.—1. What is the method of procedure when there is a remainder?—2. How must the root be expressed?

EXAMPLES.

1. The square root of 234321 is required.
Ans. 484·067†.

2. The square root of 345678 is required.
Ans. 587·94†.
3. The square root of 490023 is required.
Ans. 700·016†
4. The square root of 9432410 is required.
Ans. 3071·22†
5. The square root of 87643211 is required.
Ans. 9361·7†
6. The square root of 67432345 is required.
Ans. 8211·7†.
7. The square root of 40000000 is required.
Ans. 6324·5†

CASE II.

To extract the square root of whole numbers and decimals.

RULE.—(1) Prepare the decimals by annexing cyphers (if occasion require) so that a dot may fall on units place of the whole numbers, then proceed as in case first.

Question.—1. What is the method of extracting the square root of whole numbers and decimals?

EXAMPLES.

1. The square root of 1·4884 is required.

1·4884(1·22 root Ans.
1

22)48
44

242)484
484

2. The square root of 12·123 is required.
The decimal prepared 12·1230.
Root 3·48† Ans.
3. The square root of 9·181 is required. Ans. 3·03†.
4. The square root of 20·3331 is required.
Ans. 4·509†.
5 The square root of 11·1111 is required.
Ans. 3·3333†.

CASE. III.

To extract the square root of Vulgar Fractions.

RULE.—(1) Reduce compound fractions to simple ones, mixed numbers to improper fractions, and all to a common denominator, and also to its lowest terms: then (2) extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

If the fraction be a surd, that is (3) such an one whose root can never exactly be found, (4) reduce it to a decimal, and extract the root.

Questions.—1. What is the 1st step to extract the square root of Vulgar Fractions?—2. What is the 2d step?—3. What is a *surd?*—4. How do you extract the square root of a surd?

EXAMPLES.

1. The square root of $\frac{3044}{6849}$ is required.

$$\frac{3044}{6849} = \sqrt[2]{\frac{4}{9}} = \frac{2}{3} \text{ Ans.}$$

2. The square root of $\frac{486}{726}$ is required.

$$\frac{486}{726} = \sqrt[2]{\frac{81}{121}} = \frac{9}{11} \text{ Ans.}$$

3. The square root of $\frac{294}{1014}$ is required. $\frac{7}{13}$ Ans.

SQUARE ROOT APPLIED.

CASE I.

To form a square from any number, and to know how many can be upon a side.

RULE.—(1) The square root of the number given, will be the number upon the side of the square.

Question.—1. When a square is to be formed from any given number, how do you find the number that can be upon a side?

EXAMPLES.

It is required to lay out 25600 square rods of land in a square: I demand the side of the square that will contain the land.

$$\sqrt[2]{25600} = 160 \text{ rods on a side.}$$

2. A gentleman purchased 3025 tiles, for the purpose of paving a square yard: I demand the number that can be upon a side.

$$\sqrt[2]{3025} = 55 \text{ upon a side.}$$

8. A certain General commanded an army of 49284 men, and the better to secure his standard, he gave orders to form into a square body 4 feet distant: I demand the number of men upon a side; and also the quantity of land they occupied.

Ans. 222 men, occupy 17 acres $150\frac{2}{4}\frac{8}{1}\frac{5}{2}\frac{0}{2}\frac{}{5}$rds.

CASE II.

Square root applied in finding the area and diameter of circles by having the area and diameter given.

RULE.—(1) Divide the area by ·7854, the square root of the quotient is the diameter.

(2) Square the diameter, and multiply the square by ·7854 the product is the area.

Questions.—1. How do you find the diameter of a circle when the area is given?—2. If the diameter be given how do you find the area?

EXAMPLES.

1. I demand the length of a rope to be tied to a horse's neck that he may graze upon 7854 square feet of new feed every day, for 4 days; one end of the rope being each day fastened to the same stake.

First circle will contain 7854 sq. ft.
Second circle will contain 15708 sq. ft.
Third circle will contain 23562 sq. ft.
Fourth circle will contain 31416 sq. ft.

sq. ft. *diam.* *ft.*

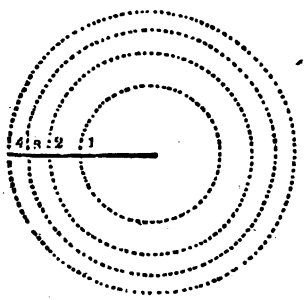
$7854 \div \cdot 7854 = \sqrt[2]{10000} = 100 \div \frac{1}{2} = 50$ first rope.

$15708 \div \cdot 7854 = \sqrt[2]{20000} = 141\dagger \div \frac{1}{2} = 70 \cdot 5$ sec. rope.

$23562 \div \cdot 7854 = \sqrt[2]{30000} = 173\dagger \div \frac{1}{2} = 86 \cdot 5$ third rope

$31416 \div \cdot 7854 = \sqrt[2]{40000} = 200 \div \frac{1}{2} = 100$ fourth rope.

NOTE.—The following figure will serve to illustrate the idea; each ring in the figure contains an equal area, viz. 7854 square feet.



The first day the horse will graze from the centre to 1; the second day to 2; the third day to 3; and the fourth day to 4; and will each day graze on the same quantity of new feed.

2. Required the difference between the area of a circle, whose radius (or semi diameter) is 50 yards, and its greatest inscribed square? Ans. 2854 yards.

3. A. B. C. and D. purchased a large grindstone, the diameter of which was 200 inches; they agreed that D. should wear off his share first, and that each man should have it alternately till they had worn off their shares; how much must each man wear off round the stone?

Ans. D. 13½; C. 16; B. 20½; A. 50 inches.

NOTE.—*Rule for the operation of the above.*

Divide the square of the diameter by the number of men, subtract the quotient from the square, and extract the square root of the remainder, which is the length of the diameter, after the first man has ground his share; this work being repeated by subtracting the same quotient from the remainder, for every man to the last; extract the square root of the remainders, and subtract those roots from the diameters, one after another; the several remainders will be the answer.

NOTE.—The preceding figure represents the grindstone, and the four rings each man's share in the same.

CASE III.

Square root applied in finding sides to triangles.

THEOREM. FIRST.

The base and perpendicular given to find the hypothenuse.

RULE.—(1) Square the base and perpendicular, add the

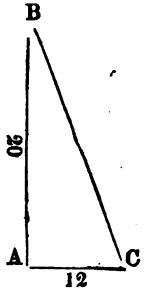
two squares together, extract the root of their sum: the root is the hypothenuse.

Question.—1. When the *base* and *perpendicular* are given, how do you find the *hypothenuse?*

EXAMPLES.

1. A man wishes to make a ladder, that will reach from the eaves of his house, to the ground, 12 feet from the house; the eaves of the house are 20 feet from the ground; I demand the length of the ladder.

B. A. and A. C. given to find B. C.



$20 \times 20 = 400$

$12 \times 12 = 144$ ft.

$\sqrt{544} = 23{\cdot}3\dagger$ B. C. Ans.

THEOREM SECOND.

The base and hypothenuse given to find the perpendicular.

RULE.—(1) Square the base and hypothenuse; subtract the least from the greatest; extract the root of the remainder; the root is the answer, or perpendicular.

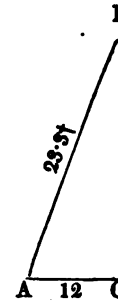
Question.—1. When the *base* and *hypothenuse* are given, how do you find the *perpendicular?*

EXAMPLES.

1. What is the height of the eaves of a house that requires a ladder 23·3† feet long to reach from the ground to the same; allowing the bottom of the ladder to stand 12 feet from the bottom of the house?

A. B. and A. C. given to find C. B.
A. B. $23{\cdot}3 \times 23{\cdot}3 = 542{\cdot}89\dagger$
A. C. $12 \times 12 = 144{\cdot}$

$\sqrt[2]{398{\cdot}89} = 19{\cdot}9\dagger$, or 20 Ans.



THEOREM THIRD.

The hypothenuse and perpendicular given, to find the base.

RULE.—(1) Square the hypothenuse and perpendicular, subtract the less from the greater, extract the root of the remainder: the root is the answer, or base.

Question.—1. When the *hypothenuse* and *perpendicular* are given, how do you find the *base?*

EXAMPLES.

1. Hypothenuse 23·3†; perpendicular 20; required the base. Ans. 11·9†, or 12 ft.

CUBE ROOT.

DEFINITION.—A cube is (1) the third power of any number: *that is*, any number multiplied by itself, and that product again by the same number, produces a Cube; thus (2) $9 \times 9 = 81 \times 9 = 729$ the cube; and 9 is the root.

EXTRACTION OF THE CUBE ROOT.

The extraction of the cube root is (3) the finding of a number which has been multiplied three times by itself.

RULE.—(4) Separate the number into periods of three figures each, beginning in units place; thus, $94\dot{3}21\dot{3}42\dot{7}$; (5) find the greatest cube in the first period, place the root in the quotient, and its cube under the first period; subtract it

therefrom, and to the remainder bring down the next period, which call the dividend, or resolvend; (6) find a divisor by multiplying the square of the quotient by 300, seek how many times the divisor is contained in the resolvend, and put the answer in the quotient.

To find the subtrahend.

(7) Multiply the divisor by the last quotient figure, and place the product under the last dividend (units under units, &c.:) then (8) square the last quotient figure, and multiply the square by the preceding figures in the quotient, and the last product multiply by 30, and place this product also under the dividend (units under units, &c.:) then (9) cube the last quotient figure, and place its cube under the dividend, also (units under units, &c.) (10) add the three numbers (which have been placed under the dividend) together, and subtract their sum from the last dividend; and (11) to the remainder bring down the next period of figures, and proceed as before, until all have been brought down, and the quotient is the root.

PROOF.—(12) Cube the quotient, or root, add in the remainder (if there is any) and the sum will be like the given cube if the work is right.

Questions.—1. What is a cube?—2. Give an example.—3. What is the extraction of the *cube root?*—4. What is the 1st step in the operation?—5. What the 2d?—6. What the 3d?—7. What the 4th?—8. What the 5th step?—9. What the 6th?—10 What then do you do?—11. How do you then proceed?—12. What is the method of proof?

EXAMPLES.

1. The cube root of 33076161 is required.

3×3×3=27)33076161(321 root Ans.
27

1st Divis. 3×3×300=2700)6076 1st divid. or resolvend.

5400=2700 × 2 =5400
360=2 ×2 ×3 ×30= 360
8=2 ×2 ×2 = 8

5768 the 1st subtrah'd 5768

2d Di. 32×32×300=307200)308161 second dividend.

307200=307200×1=307200
960=1×1×32×30=960
1=1×1×1 = 1

308161 second subtra. 308161

.....

NOTE.—In the above example I first found the greatest cube that was in the first period, viz. 27 and its root was 3, which I put in the quotient, and the cube I put under the first period, and subtracted it therefrom; and to the remainder (6) I brought down the next period of figures, viz. 076; which completed my first dividend or resolvend. Then for the first divisor, I squared the quotient or root, and multiplied the square by 300, which gave me 2700 for the first divisor; I found my divisor was contained in the dividend but twice, which I put in the quotient. Then for the subtrahend I multiplied my last divisor, viz. 2700, by the last figure in the quotient, viz. 2, which product I put under the last dividend; and I also squared the last figure in the quotient, viz. 2, and multiplied its square by the preceding figure in the quotient, viz. 3, and that product again by 30, and put this last product under the dividend; I then cubed the last figure in the quotient, viz 2, and put its cube also under the dividend: I then found the sum of the three last numbers (standing under the dividend) and subtracted their sum from the dividend; and to the remainder, viz. 308, I brought down the next and last period, viz. 161, and proceeded as before.

2. The cube root of 1728000 is required. Ans. 120.
3. The cube root of 997002999 is required. Ans. 999.
4. The cube root of 367061696 is required. Ans. 716.

NOTE.—When there is a remainder after the periods of figures are brought down, annex periods of cyphers (three at once) and point the root for decimals, and proceed as far as you please in decimals; or you may find a denominator to the remainder by the following *Rule*, viz. Square the root and multiply the square by 3; then multiply the root by 3; add the two products together, and their sum is a denominator to the remainder, which fraction must be annexed to the root to make it complete.

CASE II.

When there are decimals annexed to integers.

RULE.—(1) Prepare the decimals that a point may fall upon units in the integers, then proceed as in case first.

Question.—1. How do you extract the *cube root* when there are whole numbers and decimals?

EXAMPLES.

1. The cube root of 1780·360128 is required.
Ans. 12·12.
2. The cube root of 1815·848 is required.
Ans. 12·2.
3. The cube root of 1·442897 is required.
Ans. 1·13.

CASE III.

To extract the cube root of a vulgar fraction.

RULE.—(1) Extract the root of the numerator, for a new numerator; and the root of the denominator for a new denominator; if the fraction be a surd, that is, one whose root cannot exactly be found, (2) reduce it to a decimal and extract the root.

Questions.—1. How do you extract the cube root of a vulgar fraction?—2. If the fraction be a surd, what is the method?

EXAMPLES.

1. The cube root of $\frac{1}{1}\frac{3}{7}\frac{3}{2}\frac{1}{8}$ is required.
$\sqrt[3]{\frac{1}{1}\frac{3}{7}\frac{3}{2}\frac{1}{8}}=\frac{1}{12}$ Ans.
2. The cube root of $\frac{729}{1000}$ is required. Ans. $\frac{9}{10}$.

CASE IV.

To extract the Cube Root of mixed numbers.

RULE.—(1) Reduce the vulgar fractions to a decimal, annex it to the integers, and proceed as in case second.

Question.—1. What is the rule for extracting the *Cube Root* of mixed numbers.

CUBE ROOT APPLIED.

CASE I.

Cube root applied in finding the solidity of globular figures, by having the diameter and solidity of one given.

RULE.—Globes are in proportion to one another (1) as the cubes of their diameters; therefore (2) cube the diameter of the given globe, and also of the required globe; and then say as the cube of the diameter of the given globe,

is to its solidity, so is the cube of the diameter of the required globe to its solidity.

Questions.—1. In what proportion are Globes to one another?—2. Having the diameter and solidity of a given Globe, how do you find the solidity of a required Globe?

EXAMPLES.

1. If a cannon ball 6 inches in diameter weighs 25 lb. I demand the weight of another of the like metal, whose diameter is 3 inches.

$6\times6\times6=216$ the cube of the given diameter.
$3\times3\times3=27$ the cube of the required diameter.
As 216 : 25lb. : : 27lb. : 3·12†lb. Ans.

2 If a ball of silver 12 inches in diameter is worth $600; I demand the value of another ball, whose diameter is 15 inches. $1171·87† Ans.

CASE II.

Having one side of a cubical figure given, to find the side of another, that will contain 2, 3, 4, 5, or 6 times as much as the given one.

RULE.—(1) Cube the given side, then multiply the cube by the number mentioned in the question, (if three times as large, multiply by 3, &c.) the cube root of the product will be the side required.*

Question.—1. Having one side of a cubical figure given, how do you find the side of another that will contain 2, 3, 4, 5 or 6, &c. times as much as the given one?

EXAMPLES.

1. There is a box that is 3 feet high, 3 feet long, and 3 feet wide; I demand the side of another which shall contain 4 times the quantity.

$$3\times3\times3=27\times4=\sqrt[3]{108}=4\cdot7 \text{ feet nearly, Ans.}$$

2. There is a box that is 4 feet wide, 4 feet high, and 4 feet long; I demand the side of another that shall contain $\frac{1}{8}$ of the quantity. Ans. 2 feet.

* NOTE.—If it is required to make one which is 2, 3, 4, 5 or 6, &c. times less than the given one: divide the cube of the given side by the number mentioned in the question, and extract the root of the quotient; the root is the answer sought.

PROBLEM III.

To find the side of a cubical box that shall contain a quantity equal to any given solidity.

RULE.—(1) The cube root of the given solidity is the side of a box that will contain the same quantity.

Question.—1. How do you find the side of a cubical box that shall contain a quantity equal to any given solidity?

EXAMPLES.

1. There is a cylindrical cistern that contains 3204 solid inches; I demand the side of a cubical box, that shall contain the same quantity.

$\sqrt[3]{3204}=14{\cdot}74\dagger$ in. Ans.

2. The side of a cubical cistern, that shall hold just as much liquor as a cask whose solid content is 25519·1196 inches, is required.

$\sqrt[3]{25519{\cdot}1196}=29{\cdot}44\dagger$ in. Ans.

REDUCTION OF COINS.

The former currency of New-Hampshire, Vermont, Massachusetts, Connecticut, and Rhode-Island, reduced to dollars, cents, &c.

When the sum to be reduced is pounds only.

RULE.—(1) Annex a cypher to the pounds, divide by 3, the quotient is dollars; if there is a remainder, annex three cyphers, continue dividing, the quotient will be dimes, cents, and mills; 6 shillings is equal to $\frac{6}{20}=\frac{3}{10}$ £1×10÷3=3·333.

Question.—1. How do you reduce the former currency of the New-England States to dollars, cents, and mills, when the sum is pounds?

EXAMPLES.

1. It is required to reduce 621 pounds, into dollars, &c.

6210÷3=$2070 Ans.

2. It is required to reduce £19 into dollars, dimes, cents and mills. £190·000÷3=$63·333† Ans.

3. It is required to reduce £121 to dollars, dimes, cents, &c. Ans. $403·333†.

4. It is required to reduce £16 to dollars, dimes, cents, &c. Ans. $53·333†.

When the sum to be reduced consists of pounds, shillings, pence and farthings.

RULE.—(1) To the pounds annex half the number of shillings, and two cyphers in decimals, if the shillings are even; if they are odd, annex the greatest even half, and 5 tenths and one cypher in decimals, and divide by 3, the quotient is cents; if there are pence and farthings in the sum, (2) reduce the pence to farthings, observing to increase the sum by 1 if it exceed 12, and by 2 if it exceed 37; add the farthings, thus increased, in the place of tenths and hundredths, divide the whole by 3, the quotient is cents.

Questions.—1. If there are *shillings* in the given sum, what is the method of operation?—2. If there are *pence* and *farthings* in the sum, how do you proceed?

EXAMPLES.

1. Reduce £3 10s. 11½d. to dollars, cents, and mills.

35·00
·48=11½d.

35·48÷3=$11·826† Ans.

2. Reduce £17 19s. 11½d. to dollars, cents, and mills.

179·50
·49

179·99÷3=$59·999† Ans.

To change dollars, cents, &c. to the former currency of New-Hampshire, Vermont, Massachusetts, Rhode-Island, and Connecticut.

RULE.—When the sum to be reduced is dollars; (1) multiply the dollars by 3, double the right hand figure of the product for shillings, the remaining figures are pounds. If the sum to be reduced is dollars, cents, and mills, (2) multiply the whole by 3, point off 4 figures from the right for decimals; the rest of the product is pounds; (note, if mills are not named in the sum, three figures only must be pointed off;) find the value of the decimal part (3) by multiplying by 20, 12 and 4; observing (4) to point off for decimals each time of multiplying; and the sums standing on the left of the separatrix will be (5) shillings, pence and farthings.

Questions.—1. When the sum is dollars, how do you change it into the

former currency?—2. If the sum to be reduced is dollars, cents, and mills what is the method?—3. How is the value of the decimal found?—4. What is to be observed?—5. What will those sums on the left of the separatrix be?

EXAMPLES.

1. It is required to reduce $251 to lawful money.
   251×3=£75 6s. Ans.
2. It is required to reduce $9529 to lawful money.
   9529×3=£2858 14s. Ans
3. It is required to reduce $192 to lawful money.
   192×3=£57 12s. Ans.
4. It is required to reduce $999 to lawful money.
   £299 14s. Ans.
5. It is required to reduce $99·67 cts. to lawful money.

   99·67
   3

   29·901
   20

   18·020
   12

   ·240 £291. 18s. $\frac{3}{10}\frac{4}{0}$ Ans.

6. It is required to reduce $67·90 to lawful money.
   Ans. 20l. 7s. 4¼d.
7. It is required to reduce $99·999 to lawful money.
   Ans. 29l. 19s. 11¾†d.
8. It is required to reduce $61·276 to lawful money.
   Ans. 18l. 7s. 7¼†d.

To reduce the former currency of New-York and North Carolina to dols. cts. &c. and the contrary.

RULE.—(1) Prepare the sum to be reduced exactly as in New-Hampshire, Massachusetts, &c. and divide by 4 instead of 3, the quotient is cents; a dollar in New-York is equal to (') 8s. $\frac{8}{2}\frac{d}{0}=\frac{4}{10}$ £1×10÷4=$2·50.

Questions.—1. What is the method of reducing the former currency of New-York and North-Carolina to dols. cts. &c.?—2. What is a dollar in New-York equal to?

EXAMPLES.

1. Reduce £9 16s. to dollars, and cents, &c.
   98÷4=$24·50 Ans.

2 Reduce £230 10s. to dollars, and cents, &c.

2305÷4=$576·25 Ans.

3. Reduce £219 15s. 4½d. to dollars, cents, &c.

2197·50
·18

2197·68÷4=$549·42 Ans.

4 A merchant in Boston purchased 1200 barrels of flour in New York at £2 10s. per barrel; paid freight at 33 cts. per barrel; what does it stand him at? Ans. $7896.

5. Reduce £419 10s. to dollars, cents, &c.

$1048·75 Ans.

Change dollars, &c. to New-York and North-Carolina currency.

RULE.—(1) Proceed as in New-Hampshire currency, &c. only multiply by 4 instead of 3; the value of a dollar is equal to four tenths of a pound.

Question.—1. How do you change dollars &c. to New-York and North-Carolina Currency?

EXAMPLES.

1. Change 842 dols. to New-York currency, &c.

842
4

£336 16s. Ans.

2. Change $114·25 to North-Carolina currency, &c.

Ans. £45 14s.

New-Jersey, Delaware, Pennsylvania, and Maryland currencies reduced to the present currency of the United States, and the contrary.

RULE.—(1) Multiply the given sum (if it is pounds only,) by 8, and divide the product by 3; the quotient is dollars. If the sum is pounds, shillings and pence, (2) reduce the whole to pence, and add one ninth of the whole to itself, the sum will be cents; a dollar in these States is equal to (3) 7s. 6d., or ⅘ of a pound. 7s. 6d.=90d. $\frac{90}{240}=\frac{3}{8}$. £1×8÷3= $2·666.

Questions.—1. How do you reduce New-Jersey, Delaware, Pennsylvania, and Maryland currencies to the present currency of the United-States

when the sum is pounds only?—2. What is the method of reduction when the sum is pounds, shillings, and pence?—3. What is a dollar in these States equal to?

EXAMPLES.

1. It is required to reduce £942 to dollars, &c.

942
8

3)7536

$2512 Ans.

2. It is required to reduce £961 19s. 10d. to dollars, &c.
Ans. $2565·31 cts.

Reduce dollars, &c. to the former currency of New-Jersey, Delaware, Pennsylvania, and Maryland.

RULE.—(1) If the sum is dollars only, multiply by 3, and divide by 8, the quotient is pounds; if the sum is dollars and cents, subtract one tenth of the whole sum from itself, the remainder is pence.

Question.—1. What is the rule for reducing dollars, &c. to the former currency of New-Jersey, Delaware, Pennsylvania, and Maryland?

EXAMPLES.

1. Reduce 628 dollars to Delaware currency, &c.
$628 \times 3 \div 8 = £235$ 10s. Ans.
2. Reduce 2512 dollars to New-Jersey currency.
Ans. £942.
3. Reduce $2565·20 to Maryland currency, &c.

$\frac{1}{10}$)2565·20
256·52 subtract.

12)230868

20)19239

£961 19s. Ans.

4. A merchant of Portsmouth purchased 545 barrels of beef at $12·50 per barrel, and sent it to Philadelphia and sold it there for £5 10s. per barrel; deducted the freight at 80 cts. per barrel, and laid out the remainder of the money in flour, at £3 10s per barrel, and sent it to Portsmouth,

and paid 30 cts. per barrel freight; and sold the flour in Portsmouth for 11¼ dollars per barrel; how much flour did he purchase, and did he gain or lose by the bargain, and how much? Ans. Purchased 831 $\frac{3}{17}$ bar. flour.
Gained \$2351·79$\frac{9}{17}$.

South Carolina and Georgia currency reduced to dols. cts. and the contrary.

RULE.—If the sum to be reduced is pounds, (1) multiply by 30, and divide by 7, the quotient is dollars; (a dollar in South Carolina is equal to (2) 4s. 8d. or seven thirtieths of a pound;) if there are shillings and pence, &c. reduce the whole to pence, annex two cyphers and divide by 56, the quotient is cents. 4s. 8d.=56d.=$\frac{56}{240}$=$\frac{7}{30}$; £1×30÷7 =\$4·28$\frac{4}{7}$.

Questions.—1. What is the rule for reducing South Carolina and Georgia currency to dolls. cts. &c. when the sum is pounds?—2 What is a doll equal to in South Carolina? 3 What is the method of reduction, when there are shillings and pence in the sum?

EXAMPLES.

1. It is required to reduce £56 to dollars, &c.

56
30

7) 1680

\$240 Ans.

2. It is required to reduce £21 to dollars, &c.
Ans \$90.

3. Reduce £22 9s. 4d to dollars, &c. Ans \$96·28$\frac{4}{7}$.

Dollars, &c. reduced to Georgia and South Carolina currency.

RULE.—(1) Multiply dollars by 7, and divide by 30, the quotient is pounds; if there are cents in the sum, multiply by 56, reject two figures from the right of the product, the rest of the figures are pence.

Question.—1. How do you reduce dolls. &c. to Georgia and South Carolina currency?

EXAMPLES.

1. Reduce \$240 to Georgia currency, &c.
240×7÷30=£56 Ans

2. Reduce $90 to South Carolina currency, &c.

$90 \times 7 \div 30 = £21$ Ans.

3. Reduce $96·28\frac{1}{2}$ to Georgia currency.

$96·28\frac{1}{2}$
56

57768
48140
32

12)5392·00

449s. 4d.=£22 9s. 4d. Ans.

To reduce Canada and Nova Scotia currency to the present currency of the United States, and the contrary.

RULE.—When the sum to be reduced is pounds, (1) multiply by 4, the product is dollars, (1 dollar is equal to (2) 5 shillings in Canada, &c.;) if shillings and pence are in the sum, (3) reduce the whole to pence, annex 2 cyphers, and divide by 60, (note, 60 pence are equal to a dollar,) the quotient is cents. $\frac{5}{2}\frac{0}{0} = \frac{1}{4}$.

Questions.—1. How do you reduce Canada and Nova Scotia currency to the present currency of the United States, when the sum is pounds?—2. What is a dollar in Canada equal to?—3. If shillings and pence are in the sum, how do you then proceed?

EXAMPLES.

1. Reduce £108 to dollars. $108 \times 4 = \$432$ Ans.
2. Reduce £121 to dollars. $121 \times 4 = \$484$ Ans.
3. Reduce £1057 5s. Canada currency, to dollars, &c.

1057 : 5
20

21145
12

253740·00÷60=$4229·00 Ans.

Dollars, &c. reduced to the currency of Canada and Nova Scotia.

RULE.—(1) If the sum to be reduced is dollars, divide them by 4, the quotient is pounds; if there are dollars and

cents, multiply by 60, rejecting two figures on the right, the product is pence.

Question.—1. What is the method of reducing dollars, &c. to the currency of Canada and Nova Scotia?

EXAMPLES.

1. Reduce $432 to Canada currency, &c.
   432÷4=£108 Ans.
2. Reduce $484 to Canada currency, &c.
   484÷4=£121 Ans.
3. Reduce $4229·20 to Canada currency, &c.

   422920
   60

   12)253752|00

   21146÷20=£1057 6s. Ans.

The present currency of England reduced to the present currency of the United States.

*When the sum is pounds only.**

RULE.—(1) Multiply the pounds by 4·44 and point off 2 figures from the right, for cents, the rest are dollars.

Question.—1. What rule have you for reducing the present currency of England to the present currency of the United States?

EXAMPLES.

1. Reduce £16 sterling to dollars.
   16×4·44=$71·04 cts. Ans.
2. Reduce 223 pounds sterling to dollars, &c.
   Ans. $990·12.

If the sum consist of pounds and shillings.

RULE.—(2) To the pounds annex half the number of shillings, in the place of tenths; multiply the whole by 4·44, and point off three figures from the right, for cents and mills, the rest are dollars: If the shillings are odd, (3) annex the greatest even half and for the odd shilling annex

* In England, accounts are kept in pounds, shillings, pence, and farthings, sterling; 4 farthings make 1 penny; 12 pence 1 shilling; 20 shillings 1 pound, sterling.

5 in the place of hundredths, multiply by 4·44, and point off 4 figures for cents, &c. the rest are dollars.

Questions.—1. What is the rule for reducing the present currency of England to the currency of the United States, when the sum is pounds?—2. If the sum consists of pounds and shillings, what is the method?—3. If the shillings are odd, how do you proceed?

EXAMPLES.

1. Reduce £19 19s. to dollars, &c.
19·95×4·44=$88·5780 Ans.
2. Reduce £127 16s. to dollars, &c. Ans. $567·432.

ARITHMETCAL PROGRESSION.

Any rank of numbers are said to be in Arithmetical Progression (1) when they increase by a common excess or decrease by a common difference; (2) such as the numbers 1, 2, 3, 4, 5, &c. and 7, 5, 3, 1.

The numbers which form the series are called the (3) terms. These are (4) five in number, (5) three being given, the other two may be readily found.

1. The first term,
2. The last term, } commonly called the (6) extremes.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

CASE I.

The first term, the number of terms, and the common difference being given, to find the last term.

RULE.—(7) Multiply the number of terms less 1, by the common difference, to that product add the first term, the sum will be the last term.

Questions.—1. When are any rank of numbers said to be in Arithmetical Progression?—2. Give an example.—3. What are the numbers called which form the series?—4. How many terms are there?—5. How many are given?—6. What are the first and last terms commonly called?—7. When the first term, the number of terms, and the common difference are given, how do you find the last term?

EXAMPLES.

1. The first term is 3, number of terms 7, and common difference 5, what is the last term? 6×5+3=33 Ans.

2. A traveller went 6 miles the first day, 9 the second, &c. increasing every day's journey 3 miles, he travelled 61 days; how many miles did he go the last day? Ans. 186.

CASE II

The first term, the last term, and the number of terms being given, to find the aggregate, or total sum of all the terms.

RULE.—(1) Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

Question.—1. How do you find the aggregate or total sum of all the terms?

EXAMPLES.

1. The first term is 2, the last term 53, and the number of terms 18; required the sum of the series.

$53+2\times18\div2=495$ the Ans.

2. A traveller went 6 miles the first day, 186 the last day; he travelled 61 days; how many miles did he go in all?

$6+186\times61\div2=5856$ miles, Ans.

To answer the following questions, find the last term by case 1, *and the sum of all the terms by case* 2.

1. A merchant sold 100 yards of cloth, the first yard for 1s. the second 2s. the third 3s. &c. how much did he receive for the cloth? Ans. £252 10s.

2. Bought 19 yards of shalloon, at 1d. for the first yard, 3d. for the second, 5d. for the third, &c., increasing 2d. every yard; what did I give for the whole?

Ans. £1 10s. 1d.

3. A mercer sold 20 yards of silk, at 3d. the first yard, 6d. the second, 9d. the third, &c.; I demand what he sold the 20 yards for? Ans. £2 12s. 6d.

4. If 100 stones be placed in a right line, exactly 1 yard asunder, and the first one yard from a basket, what length of ground will the man go who gathers them up singly, returning with them one by one to the basket?

Ans. 5 miles, 1300 yards.

CASE III.

The first term, the last term, and the number of terms being given, to find the common difference.

RULE.—(1) Divide the difference of the extremes by the

number of terms less 1, and the quotient will be the common difference.

Question.—1. How do you find the common difference?

EXAMPLES.

1. The extremes are 2 and 53, and the number of terms 18, required the common difference? Ans. 3.

2. A man is to travel from Boston to a certain place in 12 days, and to go 3 miles the first day, and increasing every day by an equal excess, so that the last day's journey may be 58 miles; required the daily increase, and the distance of the place from Boston?

Ans. daily increase 5m. distance 366m.

CASE IV.

Given the first term, the last term, and the common difference, to find the number of terms.

RULE.—(1) Divide the difference of the extremes by the common difference, and the quotient increased by one is the number of terms required.

Question.—1. The first and last terms, and common difference being given, how do you find the number of terms?

EXAMPLES.

1. If the extremes be 3 and 19, and the common difference 2, what is the number of terms? Ans. 9.

2. A man going a journey, travelled the first day 5 miles, the last day 35 miles, and increased his journey every day 3 miles, how many days did he travel? Ans. 11.

GEOMETRICAL PROGRESSION.

(1) Any series of numbers, the terms of which gradually increase or decrease by constant multiplication, or division, is said to be in geometrical progression; (2) thus, 4, 8, 16, 32, 64, &c., and 243, 81, 27, 9, 3, 1, &c., are series in geometrical progression, (3) the one increasing by a constant multiplication by 2, and the other decreasing by a constant division by 3.

The number by which the series is constantly increased or diminished, is called (4) the ratio.

CASE I.

Given the first term and the ratio, to find any other term assigned.

RULE.—(5) Set down the first number, and multiply it by the ratio, and that product again by the ratio, and thus go on for 5, 6 or 7 terms, at pleasure: then multiply any of those places by itself, and divide the product by the first number, and it will be the double of that, wanting one place.

Questions.—1. What series of numbers are said to be in *geometrical progression*?—2. Give an example.—3. Why are these considered in geometrical progression?—4. What is the number called by which the series are constantly increased?—5. If the *first term* and *ratio* are given, how do you find any other term assigned?

EXAMPLES.

1. The first term of a geometrical series is 2, number of terms 13, and the ratio 2; required the last term.

1, 2, 3, 4, 5, 6, 7,
2, 4, 8, 16, 32, 64, 128, leading term.

Then 128×128=16384, which divided by 2, the first number gives 8192 the answer.

2. Required the 12th term of a geometrical series, whose first term is 3 and ratio 2.—

1, 2, 3, 4, 5, 6,
3, 6, 12, 24, 48, 96,

Then 96×96=9216÷3=3072=11th term, which,* ×2=6144=12th term the answer.

3. The first term of a geometrical series is 1, the ratio 2, and the number of terms 23, required the last term.

1, 2, 3, 4, 5, 6.
1, 2, 4, 8, 16, 32. Then 32×32=1024=11th term, which, ×2=2048=12th term, then, 2048×2048=4194304=23 term the answer.

CASE II.

Given the first term, the last term, and the ratio, to find the aggregate or total sum of the series.

RULE.—(1) Multiply the last term by the ratio, and from the product subtract the first term, and the remainder, di-

* If the ratio had been 3, 4, or 5, &c. you must have multiplied by it.

vided by the ratio less one, will give the sum of the whole series.

Question.—1. The first term, the last term, and ratio being given, how do you find the total sum of the series?

EXAMPLES.

1. The first term of a series in geometrical progression is 1, the last term is 2187, and the ratio 3, what is the sum of the series? $2187\times3=6561-1=6560\div2=3280$ Ans.

2. The extremes of a geometrical progression are 1, & 65536, and the ratio 4, what is the sum of the series?

Ans. 87381.

To answer the following questions, find the last term by case 1, *and the sum of the series by case* 2.

1. A merchant sold 15 yards of satin, the first yard for 1s. the second for 2s. the third for 4s. the fourth for 8s. &c. I demand the price of the whole? Ans. £1638 7s.

2. A draper sold 20 yards of cloth, the first yard for 3d. the second for 9d. the third for 27d. &c. in triple proportion geometrical; required the price of the whole?

Ans. £21792402 10s.

3. A crafty servant agreed with a farmer (ignorant in numbers) to serve him 12 years, and to have nothing for his service but the produce of a wheat corn for the first year, and that product to be sowed for the second year, and so on from year to year, until the end of the said time—I demand the worth of the whole produce, supposing the increase to be in a tenfold proportion, and sold at 4s. per bushel? Ans. £45211 4s. rejecting remainders.

NOTE.—7680 wheat or barley corns are supposed to make 1 pint.

4. A thresher worked 20 days at a farmer's, and received for the first day's work 4 barley corns, for the second 12, for the third 36, and so on in triple proportion geometrical —I demand what the 20 days' labor came to, supposing the whole quantity to be sold for 2s. 6d. per bushel?

Ans. £1773 7s. 6d. rejecting remainders.

5. A cunning jockey had a fine horse, to which a gentleman took a particular fancy, and after many words had passed between them, the jockey agreed to sell him to the gentleman for the price his shoes would come to, at one farthing for the first nail, and to double the price every nail; the number of nails were 32. I demand what the horse came to at that rate? Ans. £4473924 5s. 3½d.

PART SECOND.

MENSURATION
OF
PLANES AND SOLIDS.

DUODECIMALS, OR CROSS MULTIPLICATION.

DEFINITION.—Duodecimals is a rule used much in (1) the mensuration of superficies and solids; and teaches (2) to multiply feet and inches, by feet and inches without reducing. Inches are called (3) primes, and are marked thus (′); inches are divided (4) into twelfths, or seconds, and marked thus (″); thirds, or the twelfth part of a second is marked thus (‴); thus all denominations less than a foot (5) decrease in a twelve fold proportion and are designated by these characters, inches (′), seconds (″), thirds (‴), fourths (⁗), fifths (⁗′), &c.

RULE.—Write down the number to be multiplied, (6) in feet and primes, or inches, &c., and (7) under the multiplicand write down the number to multiply by; observing (8) to write every denomination under that of the same name; (9) multiply the highest denomination of the multiplier into all the denominations of the multiplicand, and set down the whole products, &c.; then multiply the next highest denomination in the multiplier into all the denominations of the multiplicand, and remove the products one place to the right, and set them all down; proceed thus through all the denominations, observing to set down the whole products each time of multiplying; then (10) add up the several products in the order in which they stand, and carry the same as in Compound Addition, the sum will be the product required.

Questions.—1. What do you learn by *Duodecimals*?—2. What do they teach?—3. What are *inches* called?—4. How are *inches* divided?—5. Do

Artificers' work is calculated (4) by different dimensions.

Glazing (5) by the square foot.

Mason's flat work, such as plastering, (6) by the square foot or yard.

Painting, paving, &c. (7) by the yard.

Partitioning, flooring, roofing, tiling, &c. are calculated (8) by the square of 100 feet.

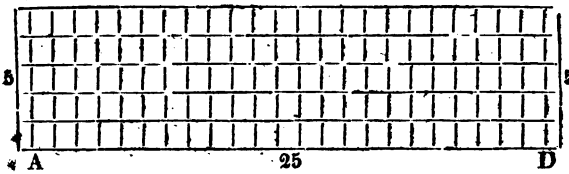
Brick work is generally calculated (9) by the solid foot.

Questions.—1. What is Superficial measure?—2. How is land measured and its dimensions taken?—3. How are boards measured and their dimensions taken?—4. How is artificers' work calculated?—5. How glazing?—6. How mason's flat work?—7. Painting and paving?—8. How partitioning, flooring, roofing, tiling, &c.?—9. How is brick work generally calculated?

CASE I.

To measure a parallelogram.

DEFINITION.—(1) A parallelogram is a figure bounded by two parallel sides which are longer than two other sides which are parallel, which four sides make four right angles, as in the following figure.



RULE.—(2) Multiply the length by the breadth, and such dimension as the length and breadth are taken in, such will be the dimension of the area; if feet, the area will be square feet, &c.

NOTE.—It is evident from the preceding figure, that if the length B C 25, be multiplied by the width A B, or D C 5, the product will be 125, and will be the number of squares contained in the figure.

Questions.—1. Describe a *Parallelogram*.—2. How do you find the area of a parallelogram?

EXAMPLES.

1. What is the area of a floor that is 22 feet 6 in. by 14 ft. 9 in.?

feet.	*in.*	
22	6	
14	9	
308	84	
	198	54′
Area 331	10	6 Ans.

2. How many feet of boards will cover a floor of a hall, that is 41 ft. 9 in. by 30 ft. 6 in., allowing the floor to be 2 boards thick? Ans. 2546 ft. 9 in.

3. How many square feet are in a board that is 21 feet long, and 11 inches wide?

Ans. 19*ft.* 3*in.* $=19\frac{3}{12}$

CASE II.

To measure a board or any other plane, when it is wider at one end than the other, and of a true taper.

RULE.—(1)Add together the width of the two ends, and half the sum is the mean width; or take the width in the middle (which is the same as the half of the sum of the width of the two ends,) then multiply the length by the mean width, the product is the answer, or area required.

Question.—1. How do you measure a board, or any other plane, when it is wider at one end than the other, and of a true taper?

EXAMPLES.

1. What is the superficial content of a board, that is 1 ft. 7 in. wide at one end, and 7 in. at the other; and 23 ft. 11 in. long?

ft.	*in.*	*in.*	*ft.*	*in.*	*ft.*	*in.*	
1	7	$+7=2$	$2\div 2=1$		1		mean width.
1	ft. 1′	$\times$ 23	ft. 11′	$=25$ ft. 10′	11″		Ans.

2. What is the area of a piece of land that is 80 rods long; 20 rods wide at one end, and 18 rods at the other?

Ans. 570 rods.

3. What is the area of a hall that is 32 ft. long; 22 ft. wide at one end, and 20 at the other? Ans. 672 ft.

4. A man has a farm lying 300 rds. on the road, and the width of it at one end is 80 rds. and at the other 60: I demand the content of the farm.

Ans. 21000 rds. or 131 acres, 40 rds.

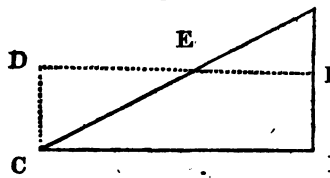
CASE III.

To measure the surface of a right angled triangle.

DEFINITION.—(1) A right angled triangle is formed by a right line falling perpendicularly on another line, as the line A B falling upon the line C B, makes a right angle at B.

A

Figure 1.

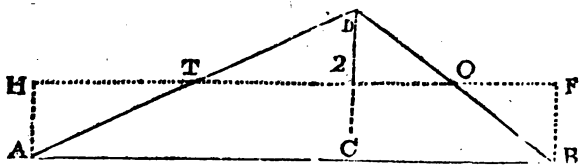


RULE.—(2) Multiply the base C B, by half the perpendicular A B, and the product will be the area; or multiply the base and perpendicular together, and half the product will be the area: also the perpendicular multiplied by half the base, the product is the area.

NOTE.—All triangles, not having one right angle, are in general terms called (3) oblique angled triangles, &c. Figure second represents an oblique angled triangle; and (4) the same rules that apply in measuring a right angled triangle will apply in measuring an oblique angled triangle.

Questions.—1. Describe the formation of a right angled triangle?—2. How do you find the area of a right angled triangle?—3. What are those angles called, that have no right angle?—4. What rules are applied in measuring oblique angled triangles?

Figure 2.



EXPLANATION.—It is evident that the area of any triangle may be found by multiplying the base by half the perpendicular; or by multiplying the base and perpendicular together, and taking half the product; or by multiplying the perpendicular by half the base. By multiplying half of A B (fig. 1) by C B, the parallelogram D F C B is measured; E F A is in the triangle

and is not measured; C D E is not in the triangle and is measured in the parallelogram; C D E being equal to E F A there can be no loss sustained; in figure 2, multiplying half the perpendicular D C, by the base A B, reduces the triangle A D B to the parallelogram H P B A: T 2 D is equal to T H A, and D 2 O is equal to O P B. Therefore it is evident that the parallelogram H P B A is equal in area to the triangle A D B.

EXAMPLES.

1. What is the content of a piece of land 13½ rods on the base, and 8·8 rods on the perpendicular, in form of figure first? Half perp. $=4{\cdot}4\times13{\cdot}5=59{\cdot}4$ rds. Ans.

2. What is the area of a piece of ground in form of figure 2d, base 27·4 rods; perpendicular 8·2 rods?

Ans. 112·34 rods.

3. What is the area of the gable end of a house, beam 40 feet, and height 12 feet 6 inches? Ans. 250 sq. ft.

4. What is the superficial content of a piece of board, in form of figure first: the length of which is 16 ft. 9 in. and the wide end 1 ft. 10 in. and the narrow end coming to a point?

ft.	*in.*	*in. ft.*	*in. ft.*	*in.*	"
1	$10\div\frac{1}{2}=11\times16$		$9=15$	4	3 Ans.

5. What is the content of the second figure, A B 20 ft. and C D 6 ft. 4 in. Ans. 63 ft. 4 in.

CASE. IV.

Board Measure.

To make a board rule, and to measure boards with the same.

Rule.—(1) Furnish a piece of hard wood of fine grain, three feet and a half long,* using six inches for a handle; plane it to as many as twelve sides; number each side or line at the lower end from 10 to 21, or from 12 to 23, as you may think proper; divide the staff into feet by driving in brass nails at every foot; divide each foot, on each line, into as many equal parts as are equal to the number on the line at the bottom; a foot on the line numbered 10 at the bottom, must be divided into ten equal parts to a foot, the line numbered 12, a foot must be divided into 12 equal parts: number these divisions on each line, beginning three or four divisions from the bottom, so that the figures on the divisions do not interfere with those on the lines at the bot-

* Any other length may be used, but I use this length as I think it most convenient.

tom; after stamping the figures, fill them up with good black ink, and give the whole a coat of varnish, and the figures will appear very plain.

To measure boards with the above rule.

RULE.—(2) Find the length of the board, then look on the end of the rule for the number corresponding to the length, apply that line of the *rule* to the middle of the board, and the width will extend on *that line* to a number expressing its area, or content.

Questions.—1. How do you make a *board rule* with which you can measure boards?—2 How can you measure boards with this *rule*?

EXAMPLE.

What is the area of a board that is 11 feet long and 12 inches wide? Ans. 11 feet.

NOTE.—The board being 11 feet long, I look for the line numbered 11 at the bottom, and apply that line of the rule to the middle of the board, and the width extends to the division on that line numbered 11, which is the content of the board.

CASE V.

To measure boards with Gunter's sliding rule.

RULE—(1) Bring the width of the board in inches, on the slider, against 12 on the line above; then look along on the line above for the number expressing the length of the board; and against that number (on the slider) stands the number expressing the area of the board.

Question.—1 How do you measure boards with Gunter's sliding rule?

EXAMPLE.

What is the content of a board that is 18 feet long and 10 inches wide? Ans. 15 feet.

NOTE.—I first brought the width of the board in inches, against 12 on the line above; I then looked on the line above for the length of the board, viz. 18; and against the length (on the slider) stood 15, the number answering to the area of the board.

CASE VI.

To measure boards with Gunter's scale and dividers.

RULE.—(1) On the line of numbers extend from 1 to the width, and that extent will reach from the length to the

superficial content or area; any right angled parallelogram may be measured by the same rule.

Question. 1.—How do you measure boards with Gunter's scale and dividers?

EXAMPLE.

What is the superficial content of a board that is 20 feet long and 1¼ feet wide? Ans. 25 feet.

NOTE.—On the line of numbers I extend from 1, to 2½ tenths, or ¼ of the distance from 1 to 2; and that extent reaches from 20, (the length of the board) to the superficial content 25.

CASE VII.

To measure joist, plank, &c.

DEFINITION.—Joists are (1) of different dimensions, sometimes 3 by 3, or 3 by 4, &c. plank are also of different thickness; and both plank and joists are reduced (2) to board, or superficial measure and are measured thus:

RULE.—(3) Find the area of one side of the joist, or plank, by any of the preceding rules in superficies; multiply that area by the thickness in inches, the last product will be the superficial content of the joist, or plank.

Questions.—1. Of what dimensions are joists and plank?—2. To what are they reduced?—3. How are they measured?

EXAMPLES.

1. What is the superficial area, or board measure of a joist, that is 20 feet long, 4 wide, and 3 thick?

20 ft. 0′×0 ft. 4′=6$\frac{8}{12}$ ft. ×3′ thick equal 20 ft. Ans.

2. What is the area or board measure of a plank that is 25 feet long, and 16 in. wide, and 3 in. thick? Ans. 100 ft.

CASE VIII.

To measure any irregular plane surface.

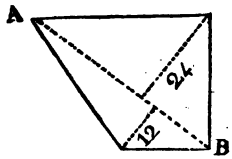
RULE.—(1) Divide the whole surface into triangles, and measure each triangle separately, as taught in case third, superficial measure.

Question.—1. What is the rule for measuring an irregular plane surface?

EXAMPLE

What is the superficial content of a plat of ground, in

form of the following figure; diagonal A B 50 feet; perpendiculars, 24 and 12? Ans. 900.



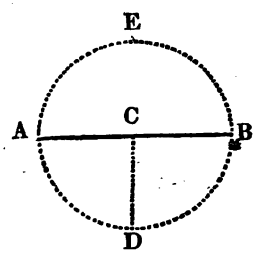
CASE IX.

To measure the surface of a circle.

DEFINITION.—Circles are (1) round figures bounded every where by a circular line called the periphery, or arch, or sometimes the circumference; the line passing through the centre is called (2) the diameter; half the diameter, or a line proceeding from the centre to the periphery is called (3) the semi-diameter, or radius.

See the figure.

A E B D is the periphery or arch.
A B is the diameter of the same.
C D is the semi-diameter or radius.



PROBLEM 1.

Diameter given to find the circumference.

RULE 1.—(4) As 7 is to 22 so is the diameter to the circumference.

RULE 2.—(5) As 113 is to 355 so is the diameter to the circumference.

Questions.—1. What are circles?—2. What is the line called which passes through the centre?—3. What is the line called, which proceeds from the centre to the periphery?—4. What is the 1st rule to find the circumference when the diameter is given?—5. What is rule 2d?

EXAMPLE.

What is the circumference of a circle, which has a diameter of 14 rods.

By Rule 1. As 7 : 22 : : 14 to 44 rods Ans.
By Rule 2. As 113 : 355 : : 14 to $43\frac{1}{113}$ Ans.*

PROBLEM II.

The circumference given to find the diameter.

RULE 1.—(1) As 22 is to 7 so is the circumference to the diameter.

RULE 2.—(2) As 355 is to 113 so is the circumference to the diameter.

RULE 3.—(3) Annex two cyphers to the circumference and divide by 3·14 the quotient is the diameter nearly.

Questions.—1. The circumference being given, how do you find the diameter?—2. What is the 2d method of finding the diameter?—3. What is the 3d rule?

EXAMPLE.

What is the diameter of a circle whose circumference is 44 rods?

By Rule 1.—As 22 : 7 : : 44 : 14 Ans.
By Rule 2.—As 355 : 113 : : 44 : $14\frac{2}{5}\frac{2}{5}$ Ans.
By Rule 3.—As 44·00 ÷ 3·14 = $14\frac{1}{7}\frac{4}{4}$ Ans.

PROBLEM III.

The diameter and circumference given to find the area.

RULE.—(1) Multiply half the diameter by half the circumference, the product is the answer; or multiply the diameter and circumference together and ¼ the product is the area.

Question.—1. How do you find the area when the *diameter* and *circumference* are given?

EXAMPLE.

What is the superficial content of a circle whose diameter is 14 rods, and circumference 44 rods?

rds. rds.
14 ÷ ½ × 44 ÷ ½ = 154 square rods, Ans.

The same question done by rule second.
14 × 44 = 616 ÷ ¼ = 154 square rods, Ans.

* The two methods do not exactly agree, the last method is nearest to the truth; an exact proportion between the diameter and circumference of a circle has not yet been discovered.

PROBLEM IV.

The diameter given to find the area.

RULE.—(1) Multiply the square of the diameter by ·7854 the product is the area.

Question.—1. The diameter being given, how do you find the *area?*

EXAMPLE.

What is the area of a circle whose diameter is 14 rods?

rds.
14×14×·7854=153·9384, Ans.

PROBLEM V.

The circumference given to find the area.

RULE.—(1) Multiply the square of the circumference by ·07958, the product is the area.

Question.—1. The circumference being given, how is the area found?

EXAMPLE.

What is the area of a circle whose circumference is 44 rods? 44×44×·07958=154·06688, Ans.

PROBLEM VI.

The area of a circle given to find the diameter.

RULE.—(1) Divide the area by ·7854 and extract the square root of the quotient; the root is the diameter sought. (See application of square root, case 2.)

PROBLEM VII.

The area given to find the circumference.

RULE.—(2) Divide the area by ·07958 and extract the square root of the quotient, and the root will be the circumference sought.

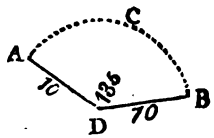
Questions.—1. When the area of a circle is given, how do you find the diameter?—2. The area being given, how do you find the circumference?

CASE X.

To measure a sector of a circle.

DEFINITION.—(1) The sector of a circle is a part of a circle bounded by an arc and two radii drawn to the ex-

tremities, see the figure A C B D. A C B the arc; A D, or D B the radii.



RULE.—(2) Find the length of the arc A C B; to do which say as 180°, is to the number of degrees in the arc, so is the radius multiplied by 3·1416, to the length of the arc; having found the length of the arc, multiply the radius by half the arc, the product is the area required.

Questions.—1. Describe the sector of a circle?—2. What is the rule for measuring a sector?

EXAMPLE.

What is the superficial content of the sector A C B D, the radius A D, or D B being 10 feet, and the arc A C B containing 135°?

As 180 : 135 : : 10×3·1416 : 23·562 *ft.* length of the arc.

then 10×23·562÷2=117·81 *ft.* Ans.

RULE 2d.—(1) Find the superficial content of a circle having the same radius; then say, as 360° (the degrees in a circle) are to the area of the whole circle; so are the number of degrees in the arc of the sector, to the superficial content of the same.

Question.—1. What is the 2d rule for finding the superficial content of a sector?

EXAMPLE.

What is the area of a sector of a circle whose arc contains 45 degrees, and the radius of the same is 20 feet?

Ans. 151 sq. ft.

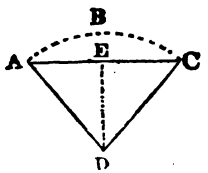
NOTE.—This method is exactly right, for it is evident that if a circle of 40 feet diameter contain 1256 square feet, that a sector of 45° with the same radius will be 1 eighth of the circle.

CASE XI.

To measure the segment of a circle.

A segment of a circle is (1) a part of a circle bounded by part of the circle's periphery, and a chord connecting the two extremities of the periphery, A B C is a segment.

See the figure.



NOTE.—The quantity of the angle A D C is 82°.

RULE.—(2) Measure the sector ABCD by case 10, and also measure the triangle ADC, and subtract the area of the triangle from the area of the sector; and the remainder will be the area of the segment ABC.

Questions.—1. What is a *segment* of a circle?—2. What is the rule for finding the area of a *segment*?

EXAMLPE.

What is the area of the segment A B C, whose arch contains 82°: and its chord A C 17·5 feet: and the perpendicular of its triangle D E 10·4 feet: and semi-diameter A D, or D C 13·7 feet?

First find the area of a circle whose diameter is 27·4 ft.

ft.

As 113 : 355 : : 27·4 : 86† circumference.

27·4×86=2356·4÷4=589·1 area of circle.

To find the area of the sector, say area.

As 360° : 589·1 : : 82° : 134·1 the area of the sector A B C D.

To find the area of the triangle.

The chord A C=17·5, perpendicular D E 10·4×17·5=182÷2=91 feet area of the triangle A D C.

Area of the sector A B C D=134
Area of the triangle A D C= 91·0

Area of the segment A B C= 43·1 ft. Ans.

CASE XII.

To find the area of a polygon.

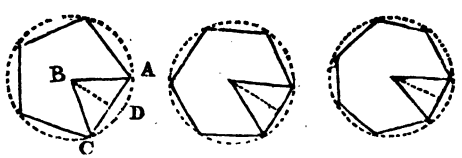
DEFINITION.—A polygon is (1) a figure having equal sides and equal angles, see figures 1. 2. 3.

RULE—(2) Multiply half the length of all the sides by the nearest distance from the centre to any of the sides; or multiply the area of one of the triangles by the number of triangles contained in the figure, the product is the answer.

Questions.—1. What is a polygon?—2. How do you find the area of a polygon?

Figures.

1 2 3



EXPLANATION.—In measuring figure first, it is evident that multiplying the distance from the centre to any side, by half the length of all the sides is measuring 5 equal triangles; the sides represent the bases, and distance from the centre, to a side of the perpendicular.

EXAMPLES.

What is the area of figure first A C equal to 8 ft.; B D equal to 5·8 ft.? $8\times5\cdot8=46\cdot4\div\frac{1}{2}=23\cdot2$ area of B A C.

There being 5 triangles contained in this figure, the area of one, viz. 23·2 multiplied by 5 will give the area of the polygon. $23\cdot2\times5=116\cdot0$ Ans.

2. What is the area of figure second, the distance from the centre to a side is 6 feet; the sides are 6, and their length 7 feet? Ans. 126 feet.

3. What is the area of figure third, the distance from the centre to a side 6·2 feet; number of sides 7; their length 5·9 feet? Ans. 128·03 feet.

CASE. XIII.

To describe and find the area of an ellipsis, or oval.

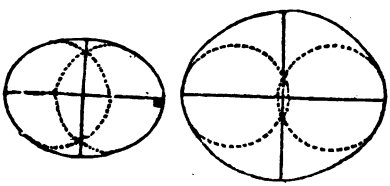
First, to describe an oval or ellipsis.

RULE.—(1) Draw a line and set one foot of the dividers on said line as a centre, and describe a circle, and move the dividers to some other point on the given line (less than the semi-diameter) and describe another circle of the same radius, and in the two points where the circle's peripheries intersect, set the dividers and complete the sides of the oval, and through these two points, draw the conjugate diameter, crossing the transverse diameter in the centre of the oval.

NOTE.—The longest diameter of an oval is called (3) the transverse, and the shortest, (4) the conjugate diameter.

Figures.

1st. *2nd.*



To find the area of an ellipsis.

RULE.—(2) Multiply the transverse by the conjugate diameter, and this again by ·7854, and the last product is the area; or multiply the two diameters together and $\frac{11}{14}$ of the product is the answer.

Questions.—1. What is the rule for protracting or describing an ellipsis or oval?—2. What rule have we for finding the area of an ellipsis?—3. What is the longest diameter of an oval called?—4. What the shortest?

EXAMPLES.

1. What is the area of figure first, the longest diameter is 17·5 and shortest 13?

17·5×13×·7854=178·67850 area Ans.

2. What is the area of figure second, the longest diameter being 21, and the shortest 17?

By rule first, 21×17×·7854=280·3878 area.

By rule second, 21×17=357÷$\frac{11}{14}$=280$\frac{7}{11}$ Ans

CASE XIV.

To find the area of a globe or sphere.

DEFINITION.—(1) A globe or sphere is bounded by a circumference every way equally distant from a point within called the centre, thus a cannon ball may be called a globe or sphere.

RULE.—(2) Multiply the diameter and circumference together, the product is the area.

Questions.—1. Describe a globe or sphere?—2. What is the rule for finding the area of a globe or sphere?

EXAMPLES.

1. What is the superficial area of a globe whose circumference is 44, diameter 14? $44 \times 14 = 616$ area Ans.

2. What is the area of the globe we inhabit, allowing it to contain 360 degrees and 69½ miles to a degree on the equator?

$360^{\circ} \times 69\frac{1}{2} = 25020$ circum. and 7964 diam. nearly.

$25020 \times 7964 = 199259280$ area in square miles.

SOLID MEASURE.

DEFINITION.—The mensuration of solids includes (1) the measuring of all bodies which have length, breadth and thickness; such as timbet, stone and wood, &c.

(2) In solid measure 1728 inches make a foot, that is 12 inches in length, 12 in breadth, and 12 in thickness; thus a solid foot would make 1728 little blocks, one inch square.

GENERAL RULE.

(3) Multiply the length, breadth and thickness together, the last product is the solidity required.

Questions.—1. What does the *mensuration* of solids include?—2. Describe a foot in solid measure.—3. What is the general rule for finding the solidity?

CASE I.

To find the solidity of a prism.

DEFINITION.—(1) A prism represents a three cornered file, that is all its length of the same bigness; therefore the ends are triangles.

RULE.-(2)Find the area of one end;* multiply this area by the length of the prism, the last product will be the solidity.

Questions.—1. Describe a prism.—2. What is the rule for finding the solidity of a prism?

EXAMPLES.

1. What is the solidity of a prism, the sides of the triangles of which measure 13 inches and the perpendiculars of its triangle 12 inches; and its length 12 feet?

in. in.

$13\times12\div\frac{1}{2}=78\times12$ feet $=936\div144=6\frac{72}{144}$ Ans.

2. What is the solidity of a prism the base of which is 2 feet 3 inches; and the perpendicular of which is 1 foot 10 inches; and length 20 feet 6 inches?

Ans. 42 feet 3′ 4″ 6‴.

CASE II.

To find the solidity of any figure that is of equal width and equal thickness.

RULE.—(1) Multiply the length, breadth and thickness together, the last product is the solidity

Question.—I. How do you find the solidity of any figure that is of equal width and equal thickness?

EXAMPLES.

1. How many solid feet are in a hall, that is 36 feet 6′ long, 14 feet 6′ wide, and 8 feet 6′ high?

36 feet 6′×14 feet 6′×8 feet 6′=4498 feet 7′ 6″, or 4498 ½ feet Ans.

2. How many solid feet will be occupied by 21 chests of tea, which are 3 feet 3 inches every way? Ans. $720\frac{5}{8}\frac{7}{4}$ ft.

CASE III.

To find the solidity of any figure that has equal thickness, but unequal width.

RULE.—(1) Find the mean width by adding together the width of both ends, and taking half the sum for the mean

* If the area of the end is found in inches, and multiplied into the length in inches, the solidity is in solid inches, and must be divided by 1728 to bring it to solid feet; if the area of one end in inches is multiplied into the length in feet, dividing by 144 will give the solidity in solid feet.

width: the width taken in the middle is also the mean width; multiply the mean width, depth and length together, and the product is the solidity.

Question.—1. How do you find the solidity of a figure that has equal thickness, but unequal width?

EXAMPLES.

1. How many solid feet are in a stone that is 21 feet long, 2 feet wide at one end, and 3 feet at the other, and 1 foot 6 inches thick. $2+3=5\div\frac{1}{2}=2{\cdot}5\times21\times1{\cdot}5=78{\cdot}75$ Ans.

2. How many solid feet in a stone wall that is 51 feet long, and 7 feet high, and mean thickness 2 ft. 6′?

Ans. 892½ feet.

CASE IV.

To find the solidity of a cylinder.

DEFINITION.—A cylinder is (1) a long round body, all its length of equal bigness.

RULE.—(2) Find the area of one end, and multiply it by the length, the last product is the solidity. To find the area of one end, apply the rule for measuring a circle. (*See Circles.*)

Questions.—1. What is a cylinder?—2. How do you find the solidity of cylinder?

EXAMPLES.

1. What is the solidity of a cylinder that has a diameter 22 inches, and is 20 feet long?

diam. circum.

$22\times69{\cdot}14=1521{\cdot}08\div\frac{1}{4}=380{\cdot}27$ area. $380{\cdot}27\times20$ ft.$=7605{\cdot}4\div144=52.81$ ft. Ans.

2. What is the solidity of a cylinder that has a diameter 9½ feet, and is 21 feet long? Ans. 1488·69.

3. What is the solidity of a cylinder, whose diameter is 7 inches, and circumference 22, and is 20 feet long?

Ans. $5\frac{5}{144}$ ft.

CASE V.

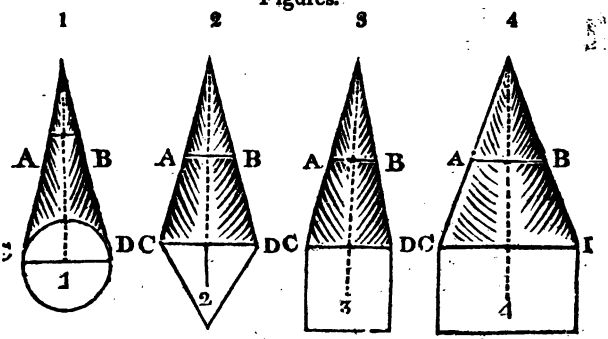
To find the solidity of a cone or pyramid.

DEFINITION.—A cone is (1) a figure standing upon a base, and of a true slant or taper from the largest end, to a point or vortex. (2) The base is either circular, square, triangular, or in the form of a parallelogram; the base of the

first figure is a circle ; the second is a triangle ; the third is a square ; the fourth is a parallelogram.

A frustum of a cone is (3) a piece cut off parallel to the base : that part between A B and C D in the following figures is called a frustum ; the length of the frustum is represented by the perpendicular line, proceeding from the centre of the base, to the line A B.

Figures.



RULE.—(4) Find the superficial area of the base, and multiply it into one third of the perpendicular height of the cone, and the product will be the solidity.

Questions.—1. What is a cone or pyramid?—2. Describe the base of a Pyramid?—3. What is the frustum of a cone?—4. How do you find the solidity of a cone?

EXAMPLES.

1. What is the solidity of figure first; perpendicular 21 feet, and the diameter of the circle at the base 9½ ft.?

9·5 diameter, 29·85 circumference.

Half diam. and half circu. 4·75×14·925=70·89† area of base.

Area 70·89×21=1488·69÷3=496·23† feet, solidity.

2. What is the solidity of figure 2d; perpendicular 21 feet, sides of its triangle 10, and perpendicular of its triangle 8½?

10×8½=85÷2=42·5×21÷3×297·5 solidity, Ans.

3. What is the solidity of figure third; perpendicular height 21, and the sides of its base 9 feet?

9×9=81×21÷4=567 ft. Ans.

4. What is the solidity of figure fourth; perpendicular height equal to 21, and sides 15 by 9½?

Ans. 997·5 ft.

CASE VI.

To find the solidity of the frustum of a cone, &c.

RULE 1.—(1)Find the superficial area of both ends, add the two areas together, and reserve the sum; multiply the two areas together, extract the square root of the product, and add the root to the reserved sum, and multiply the sum by one third the perpendicular height, the product is the solidity required.

RULE 2.—If the cone is exactly square at both ends, (2) multiply a side of the greatest, by a side of the least square; also find the difference between the two sides; square the difference; add one third of the square to the product of the two sides; multiply the last sum by the length, the product is the solidity.

Questions.—1. What is the rule for finding the solidity of the frustum of a cone?—2. If the cone be exactly square at both ends, how is the solidity found?

EXAMPLES.

1. What is the solidity of the frustum A B C D; figure first, case 5th; largest diameter 9½; smallest 8; and length 14?

Largest diam. 9·5×9·5×·7854=70·88† area largest end.
Smallest diam. 8·0×8·0×·7854=7·06† area least end.

Areas 70·88×7·06=$\sqrt[2]{500·4128}$=22·36 root added.

———len.

Ans. 100·30×14÷3=468·06†

2. What is the solidity of the frustum A B C D, figure second, case fith; the sides of its largest triangle 10, and its perpendic. 8½; sides of the smallest triangle 3·5 and the perpendicular of its triangle 8; and length 14?

10×4½=42·5 area largest end.
3·5×1·5=5·25 area smallest end.

Areas 42·5×5·25=$\sqrt[2]{223·1250}$=14·93 root added.

sum 62·68
sum 62·68×4⅔=292·5 solidity, Ans.

3. What is the solidity of the frustum A B C D, figure third, case 5th; sides of its largest square 9, and the smallest 3, and length 14?

Same question by rule second.

Sides of the two squares $9\times3=27$ product.

$9-3=6$ dif. $\times\ 6=36\div\frac{1}{3}=12$ add.

Solidity.

$39\times14=546$ Ans.

4. What is the solidity of the frustum A B C D, figure fourth, case fifth; sides of the largest end 15 by $9\frac{1}{2}$; sides of the smallest end 5 by 3·2; and 14 long?

Sides of the largest end $15\times9\cdot5=142\cdot5$ area.

Sides of the smallest end $5\times3\cdot2=16\cdot0$ area.

Areas $142\cdot5\times16=\sqrt[2]{2280\cdot0}=47\cdot7$ add.

sum 206·2

$206\cdot2\times14=2886\cdot6\div3=962\cdot2\dagger$ Ans.

5. What is the solidity of the frustum of a cone that is 10 feet square at one end, and 4 at the other, and is 32 feet long? Ans. 1664 ft.

6. What is the solidity of the frustum of a cone that is 16 in. square at one end, and 12 at the other, and is 16 feet long? Ans. $21\frac{5}{12}$

7. What is the solidity of the frustum of a cone that is 24 in. square at one end and 21 at the other, and is 10 feet long? Ans. $35\frac{5}{12}$

CASE VII.

To find the solidity of the segment of a cone, as the parts above A B *in the figures, case 5th.*

RULE.—This part forms a cone of itself, and must be measured (1) by case fifth.

Question—1. How is the solidity of the segment of a cone found? —Repeat the rule in *Case 5th.*

CASE VIII.

To find the solidity of a wedge, when the edge and large end are of equal width.

RULE.—(1) Multiply the length and width together, and that product by half the thickness, the last product is the solidity.

Question.—1. How do you find the solidity of a wedge, when the edge and large end are of equal width?

EXAMPLES.

1. What is the solidity of a wedge that is 12 ft. long, 12 wide, and 6 thick, at the large end?

$12\times12=144\times6\div4=432$ solidity, Ans.

2. What is the solidity of a wedge that is 10 ft. long, 6 inches wide, and 6 thick? Ans. $1\frac{1}{2}$ ft.

CASE IX.

To find the solidity of a wedge when the edge is narrower than the large end.

RULE.—(1) To the width of the edge add twice the width of the large end, and reserve the sum: multiply the length of the wedge by the thickness of the large end; multiply this last product and the reserved sum together, and divide by 6; the quotient is the solidity.

Question.—How do you find the solidity of a wedge, when the edge is narrower than the large end?

EXAMPLES.

1. What is the solidity of a wedge that is 12 ft. long, and 18 in. wide at the largest end, and 12 in. wide at the edge; and 6 in. thick at the large end?

Width of the edge 12
Width of the large end $18\times2=36$

48 reserved sum

Length of wedge 144 inches.
Thickness of large end 6 inches.

864 product.

Pro. $864\times48=41472\div6=6912$ in. or 4 ft. Ans.

2. What is the solidity of a wedge that is 20 ft. long, 15 wide at the large end, and 10 at the edge; and 12 thick? Ans. 1000.

NOTE.—This rule will give the solidity of a cone when it is square, or in form of a parallelogram.

EXAMPLE.

What is the solidity of the cone, figure third, case fifth; width of the large end 9, thickness 9, and length 21?

Width of the edge 0
Width of the large end 9×2=18

18 reserved sum.

21×9=189×18=3402÷6=567 Ans.

(*See Ans. to Quest. 3 Case fifth.*)

CASE X.

To find the solidity of a globe, or sphere.

DEFINITION.—A globe or sphere is (.) a round body bounded by a surface every point of which is equally distant from a point within called the centre; a line passing from one side to the other through the centre is called the diameter, or axis.

RULE.—(.) Cube the diameter, or axis, and multiply its cube by ·5236 the last product is the solidity.

Questions.—1. What is a globe or sphere? 2. What is the rule for finding the solidity of a *Globe or Sphere.*

EXAMPLES.

1. What is the solidity of a globe, or sphere whose diameter is 113?

113×113×113×·5236=7355500·8692 solidity.

2. What is the solidity of the globe which we inhabit, in solid miles; allowing its circumference to be 25000 miles?

As 22 is to 7 so is 25000 to 7954 diam. nearly. (Fractions omitted.) 263485304337 sol. miles. Ans.

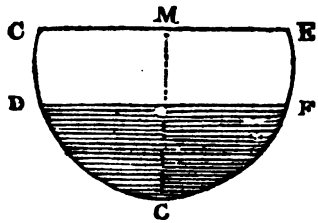
3. What is the solidity of a cannon ball that is 9 inches on its diameter? Ans. 381·7044 sol. in.

4. What is the solidity of a sphere whose diameter is 2 ft. 8 in.? Ans. 9·92† ft. solidity.

CASE XI.

To find the solidity of a segment, or part of a globe, or sphere; or part of a globe cut off parallel to the diameter, as part D C F.

See the figure.



B diam. A B. 14.

RULE.—(1) Square the radius of its base, (as D o, or F o) multiply its square by 3, and reserve the product; square the depth o C, add the square and reserved product together; and multiply the sum by the depth o C; and the last product by ·5236; the product is the solidity.

Question.—1. What rule can be given for finding the solidity of a segment or part of a globe or sphere.

EXAMPLES.

1. What is the solidity of the segment D C F, radius D o, or F o, 7; and depth o C, 5?

Radius $7\times7\times3=147$ reserved product.

Depth $5\times5=25$ add.

$172\times$depth $5=860\times\cdot5236=450\cdot296$. Ans.

2. Required the solidity of a segment of a globe, whose semi-diameter is 9 in. depth 9 in.

Ans. 1526·8176 in.

CASE XII.

To find the solidity of the middle zone of a sphere, or globe; or the part of a sphere after two segments have been cut off, parallel to the diameter or axis, as C D E F *in figure, case eleventh.*

RULE.—(1) Square the semi-diameter of both ends and add the squares together, and reserve the sum: square the height, or distance of the two ends, as o M, and add $\frac{1}{3}$ of its square to the reserved sum; multiply the sum by the height, or distance o M, and this product again by 1·5708; the last product is the solidity.

Question.—1. How do you find the solidity of the middle zone of a sphere or globe?

EXAMPLES.

1. What is the solidity of the middle zone C D E F, (case eleventh;) diameter C E, or D F, 14, and height o M, 8?

NOTE.—In this example the diameters are alike, which is not always the case.

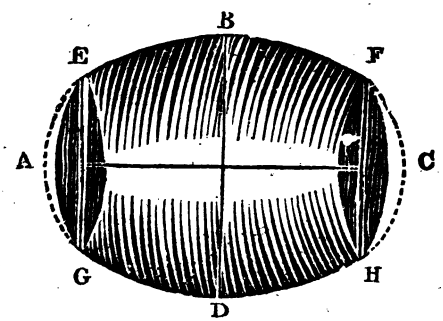
Diam. 14 semi.=7×7=49
Diam. 14 semi.=7×7=49
Height oM8×3=9+$\frac{1}{3}$= 3 add.
———*ht.*
101×3=303×1·5708=475·9524. Ans.

2. What is the solidity of the middle zone of a sphere whose greatest diameter is 12, and least 8, and height or length 10? Ans. 1340·41†

CASE XIII.

To find the solidity of a spheroid, or ellipsoid; and also to find the solidity of the middle frustum of a spheroid.

DEFINITION.—A spheroid is (1) a solid generated by the revolution of an ellipse, or oval, about the transverse of conjugate diameter. [See the figure A B C D.]



To find the solidity of the spheroid A B C D.

RULE.—(2) Multiply the revolving diameter B D, into

itself, and the product by the fixed diameter A C, and the product again by ·5236, the last product is the solidity.

Questions.—1. What is a spheroid?—2. How do you find the solidity of a spheroid?

EXAMPLES.

1. What is the solidity of a spheroid whose revolving diameter B D is 20, and fixed diameter A C is 30?
Revolv. diameter $20\times20=400\times30\times\cdot5236=6283\cdot2$ Ans.

2. What is the solidity of a spheroid whose revolving diameter is 30, and fixed diameter 50? Ans. 23562.

To find the solidity of the middle frustum of a spheroid.

RULE.—(1) To the square of the end diameter, add twice the square of the middle diameter, multiply this sum by the length, and the product again by ·2618, the last product is the solidity.

Question.—1. How do you find the solidity of the middle frustum of a spheroid?

EXAMPLES.

1. What is the solidity of the frustum E F G H, length 40, end diameters E G, or F H 24, and middle diameter B D 32?
End dia. $24\times24 = 576$
Mid. dia. $32\times32\times2=2048$ add.

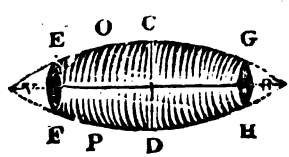
$2624\times40\times\cdot2618=27478\cdot5280$ Ans.

2. What is the solidity of the middle frustum of a spheroid, whose length is 30; middle diameter 25; and end diameter 20 inches? Ans. 12959·1.

CASE. XIV.

To find the solidity of an elliptic spindle.

DEFINITION.—(1) An elliptic spindle is formed by any of the three conic sections revolving about a double ordinate; the following figure represents an elliptic spindle.



RULE.—(2) Find a diameter half way between the middle and end, as at O P, multiply it by 2, square the product and add the square to the square of a diameter taken in the middle, multiply the sum by the length, and the product again by ·1309, the last product is the solidity, very nearly.

Questions.—1. Describe an elliptic spindle.—2. How do you find the solidity of an elliptic spindle?

EXAMPLES.

1. What is the solidity of the elliptic spindle V B C D; length 15; diameter C D 5; and diameter taken half way between the middle and end equal to 4?

Diam. O P. 4×2=8×8=64 sq. of twice O P.
Diam. C D 5×5=25 sq. of C D add.

89 sum.

Sum 89×15×·1309=174·7515 Ans.

2. What is the solidity of an elliptic spindle that is 20 long; greatest diameter 15; and the diameter half way between the middle and end, equal to 10?

Ans. 1636·25.

CASE XV.

To find the solidity of the middle frustum of an elliptic spindle.

RULE.—(1) To the square of twice a diameter taken half way between the middle and end, add the squares of the middle end diameters; multiply the sum by the length, and again by ·1309, the last product is the solidity required.

NOTE.—E F G H in the preceding figure represents the middle frustum of an elliptic spindle.

Question.—1. Give the rule for finding the solidity of the middle frustum of an elliptic spindle?

EXAMPLES.

1. What is the solidity of the middle frustum E G F H, length 10; diameter at the end 4; diameter C D 6, and diameter taken at O P equal to 5?

Diam. O P 5×2=10×10=100 sq. of twice O P.
Diam. C D 6× 6= 36 sq. of C D.
Diam. E F or G H 4× 4= 16 sq. of E F or G H.

152 sum.

Sum 152×10×·1309=198·968 Ans.

CASE XVI.

*The common, but erroneous method of finding the solidity of square timber contained in a round stick.**

RULE.—In order to ascertain the quantity of hewn timber contained in a round stick, it has been the common practice to (1) girt the stick in the middle with a line, (after taking off the bark) and then to double the line into four equal parts, and one of these parts is considered equal to a side of square timber, that can be hewn from the stick; which side is multiplied into itself in inches, and the product by the length in feet; the last product divided by 144 is considered the solidity in solid feet.

Question.—1. What is the common method of finding the solidity of square timber contained in a round stick?

EXAMPLES.

1. What solidity of square timber, can be hewn from a round stick that is 21 ft. long, and its circumference, or girt line 48 inches?

Girt $48 \div 4 = 12 \times 12 = 144 \times 21 = 3024 \div 144 = 21$ ft. Ans.

2. What is the solidity of square timber in a round stick that is 30 ft. long, and its circumference or girt measures 24 inches? Ans $7\frac{1}{2}$ ft.

3. What is the solidity of a stick of timber that girts 56 inches, and its length being 11 ft. 9 inches?

Ans. 15 ft. 11′ 11″, or $15\frac{4}{7}\frac{3}{1}$ ft.

4. What is the solidity of square timber in a stick of round timber, that girts 50 in. and is 31 ft. 7 in. long?

Ans. 34 ft. 3′ 2″ 10‴ 9⁗ or $34\frac{1}{6}\frac{3}{0}\frac{4}{1}\frac{8}{2}$ ft.

CASE XVII.

A 2nd and more accurate method of finding the solidity of square timber that is contained in a stick of round timber; or to find how much the stick will measure after it is hewn square.

RULE.—(1) Girt the stick (after taking off the bark,) and annex a cypher to the girt in inches and divide the whole by 4·4 the quotient is a side of the greatest square in inches that can be hewn from such a stick; multiply this side into

* This method gives too much for the solidity, if it is meant to measure the solidity of square timber only; and not enough if it is meant to include the four slabs, or segments.

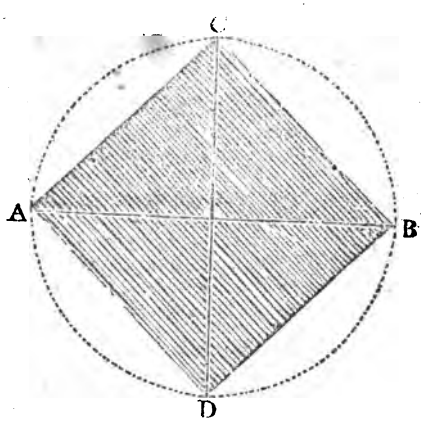
itself, and this product again by the length in feet, and the last product divided by 144 will be the solidity of the stick in feet after it is hewn.

Question.—1. What is the 2nd and more accurate method of finding the solidity of square timber contained in a stick of round timber?

Illustration of the preceding rule.

Make a circle whose diameter is 20, and of course its circumference would be 62·8† (which may represent the girt line of a stick of timber;) annex a cypher to the circumference and divide by 4·4, the quotient will be 14 3 nearly, now make the largest square in this circle that can be made in it, and you will find that the sides of the square will be 14·3 very nearly; but if the same circumference were divided by $\frac{1}{4}$, (which would represent the length of the girt line in case sixteenth, after it was doubled into four equal parts) you would find that the quotient would be 15·7 for a side of the square timber.

The following figure will further illustrate the preceding rule.



Diameter A B or C D 20, and the sides of the square A D, or D B, or B C, or C A 14·3, and are sides of the greatest square that can be made in the circle

EXAMPLES.

1. What solidity of square timber can be hewn from a stick that is 20 feet long, and girts 62·8 inches?

Girt 62·80÷4·4= 14·3 nearly × 14·3=204·49× by length 20=4089·80÷144=28·4† solidity, Ans.

The same question solved by case sixteenth.

Girt 62·8÷4=15·7×15·7=246·49× length 20=4929·80÷144=34·2† solidity Ans.

NOTE.—By comparing the two preceding answers, it is evident that the last method of operation gives too much for the solidity of square timber in such a stick.

	ft.
Solidity found by using ¼ the circumference	34·2†
True solidity found by case 17th	28·4†
difference	5·8

2. What is the solidity of square timber in a stick of round timber, that is 21 feet long, and 48 inches in circumference?

Girt 48·0÷4·4=10·9† a side of square timber in the stick. 10·9×10·9×21=2495·01÷144=17·32† solidity, Ans.

The same question solved by using ¼ of the circumference.

Girt 48÷4=12×12×21=3024÷144=21 feet, Ans.

Difference 3·68 ft. too much.

3. What is the solidity of square timber in a stick of round timber, that is 11 ft. 9′ long, and girts 56 inches?

Girt 56·0÷4·4=1 ft. 0′ 8″† a side × 1 ft. 0′ 8″=1 ft. 1′ 4″ 5‴ 4⁗ mul. by the length 11 ft. 9′=13 ft. 1′ 1″ 2‴ 8⁗ Ans.

4. What solidity of square timber is in a round stick, that is 31 ft. 7 inches long, and girts 50 inches?

50·0÷4·4=11†in.= a side. Ans. 26 ft. 6′ 5″ 7‴

NOTE.—Having shown by the two first questions in this case, that using ¼ of the circumference for a side of square timber, gives too much for the solidity, or more than the stick will measure after it is hewn square; it now remains to show that ¼ of the circumference multiplied by itself, and then by the length, does not produce the solidity of the stick, if the four segments or slabs are to be included.

EXAMPLES.

1. What is the solidity of a stick of timber whose length is 20 ft. circumference 62·8 in. and diameter, 20 in. measured by using ¼ of the circumference? Ans. 34·2† ft.

The same stick measured as a cylinder.

Ans. 43·5† ft

Exact solidity of the stick	43·5† ft.
Erroneous solidity	34·2† ft.
diff.	9·3 ft.

NOTE.—It is obvious from the preceding examples that the square of ¼ the circumference, multiplied by the length, gives too much for the solidity of square timber in the stick, (or more than it would measure if it were hewn square:) and not enough if the four slabs, or segments are to be included; It ought to be a matter of consideration by those concerned in buying and selling timber, how it should be measured; and if it is agreed by the parties to measure only the square timber, (or what the stick would measure if it were hewn) then apply the rule in case 17th; but if it is agreed to measure the whole solidity of the stick (including the segments, or four slabs,) then apply the rule in the following case.

CASE XVIII.

To find the solidity of a round stick of timber, including the four slabs, or segments.

RULE.—(1) Girt the stick in the middle (after taking off the bark,) annex two cyphers to the girt, or circumference, and divide by 3·14 the quotient is the diameter nearly: multiply the girt, or circumference and diameter together; and one fourth part of the product multiplied into the length will be the solidity required.

Question.—1. How do you find the solidity of a round stick of timber, including the four slabs or segments?

EXAMPLES.

1. What is the solidity of a stick of timber, that girts 94·2 in. and length 20 feet?

94·20÷3·14=30 diam. 94·2×30÷4=706·5×20÷144=98 $\frac{18}{144}$ ft. Ans.

2. What is the solidity of a stick of timber, that is 22 ft. long; and the girt line measuring 31·4 in.? Ans. $11\frac{44}{144}$.

NOTE.—In all the preceding examples in timber measure, the timber has been considered of equal bigness from end to end; it now remains to treat of tapering timber, both round and hewn.

CASE XIX.

To find the solidity of hewn timber in a round stick, when the stick is tapering from end to end.

RULE.—(1) Girt the stick at both ends, annex a cypher to the girts, or circumferences, and divide each girt by 4·4 the quotients will be the sides of square timber, multiply the two sides together; find the difference between the two sides; square the difference, and add one third of its square to the product of the two sides, and multiply this sum by the length, the last product is the solidity required.

Question.—1. How do you find the solidity of hewn timber in a round stick, when the stick is tapering from end to end?

EXAMPLES.

1. How much hewn timber is in a stick of round timber, that is 24 ft. long, and its circumference at one end is 44 in. and at the other 22 in.?

Cir. 44·0÷4·4=10 a side of the largest square.
Cir. 22·0÷4·4=5 a side of the least square.
Sides of the two squares 10×5=50 product.
Sides 10—5×5=25÷½ = 8½ added.

58½ sum.

Sum 58½×24ft.=1400÷144=9 $\frac{92}{144}$ ft. Ans.

2. How much hewn timber is in a stick that is 21 ft. long, and its circumference at one end is 88 in. and at the other 44 in.? Ans. 34 $\frac{4}{144}$ ft.

NOTE.—If it is required to find how much hewn timber is contained in a round stick, allowing the square timber to be all its length of equal bigness, the round stick must be girted only at the smallest end.

CASE XX.

To find the solidity of a round stick of timber which is of a true taper from end to end, including the four segments, or slabs.

RULE.—Apply (1) the rule for finding the solidity of the frustum of a round cone.

(*See Frustum of Cones, Case Sixth, Solids.*)

Question.—1. What rule do you apply for finding the solidity of a round stick of timber, which is of a true taper from end to end, slabs included?

EXAMPLE.

What is the solidity of a stick of timber whose largest circumference is 63, and smallest 43 in.; and is 21 feet long?

Cir. 63·00÷3·14=20·0† diam.
Cir. 43·00÷3·14=13·7† diam. nearly.
Cir. and diam. 63×20=1260÷4=315 area, large end.
Cir. and diam. 43×13·7=589·1÷4=147† area small end.
Areas 315×147=$\sqrt[2]{46305}$ = 215† root add.

677 sum.

Sum 677×21÷3=4739÷144=32 $\frac{131}{144}$ ft. Ans.

NOTE.—This method of operation would be too lengthy for common use; and if the following rule be adopted, the solidity may be found very near the truth (although a little too small) with much more convenience.

RULE 2.—(1) Girt the stick near the middle (but rather nearest the butt end,) the girt is the circumference; annex two cyphers to the circumference; and divide by 3·14 the quotient is the diameter nearly; multiply the circumference and diameter together, and ¼ of the product multiplied into the length will give the solidity very near.

Question.—1. What is the 2nd rule for finding the solidity of a round stick of timber which is of a true taper from end to end?

EXAMPLES.

1. What is the solidity of a round stick of timber (including the four segments) which is 12 ft. long; and its middle girt, or circumference is 62·8 inches?
Cir. 62·80÷3·14=20 diam.; and 20×62·8=1256·0÷4=314 ×12=3768÷144=26$\frac{4}{6}$ ft. Ans.

2. What is the solidity of a round stick of timber that is 10 ft. long; and its mean girt 31·4 inches, including the four segments or slabs. Ans. 5$\frac{64}{144}$ ft.

3. What is the solidity of a round stick of timber, whose mean girt is 94·2 in.; and 30 feet long? Ans. 147$\frac{72}{144}$ ft.

CASE XXI.

To find the solidity of a hewn stick of timber that has all its sides parallel.

RULE.—(1) Multiply one side by the other, and the product multiplied by the length, will be the solidity.

Question.—1. How do you find the solidity of a hewn stick of timber having all its sides parallel?

EXAMPLES

1. What is the solidity of a stick of hewn timber that is 30 ft. 6 in. long; and one side is 1 ft. 3 in.; the other side 6 in.?

30 ft. 6′×1 ft. 3′×0 ft. 6′=19 ft. 0′ 9″ or 19 $\frac{1}{16}$ ft. Ans.

NOTE.—Stone is measured in the same manner as square or hewn timber

2. What is the solidity of a stone that is 12 ft. 6 in. long, 4 ft. 10 in. wide, and 0 ft. 9 in. thick?

Ans. 45 ft. 3′ 9″.

NOTE.—When two sides of the stick are parallel, multiply the mean width and the other side together, and the product by the length.

3. What is the solidity of a stick of timber that is 12 in. wide at one end, and 8 at the other; and is 8 in. thick, and 8 ft. 4 in. long? Ans. 1 ft. 10′ 2″ 8‴.

CASE. XXII.

To find the solidity of a tapering hewn stick of timber, which has no two sides parallel.

RULE.—Apply (1) the rule for finding the solidity of the frustum of a cone or pyramid, (see case sixth;) if the stick is square at each end, (2) measure it as in case 6th, rule 2.

Questions.—1. What rule do you apply for finding the solidity of a tapering hewn stick of timber, which has no two sides parallel?—2. If the stick be square at each end, how do you measure it?

EXAMPLES.

1. What is the solidity of a stick of hewn timber that is 10 in. square at one end, and 4 in. at the other, and is 32 feet long?

Sides of sq. $10\times4=40$ pro.
Sides $10-4=6\times6=36+\frac{1}{3}=12$ add.

Sum 52

Sum $52\times32=1664\div144=11\frac{5}{9}$ ft. Ans.

2. What is the solidity of a stick of timber that is 15 in. by 12 at one end; and 9 by 6 in. at the other end, and 21 ft. long?

Sides $15\times12=180$ area of largest end.
Sides $9\times6=54$ area of smallest end.

234 sum of the two areas.

$180\times54=\sqrt[2]{9720}=98\dagger$ root add.

Sum $332\times7=2324\div144=16\frac{5}{36}$ ft. Ans.

NOTE.—The preceding method of measuring hewn tapering timber gives the exact solidity; but the method of taking the dimensions in the middle of the stick does not give the exact solidity, except in such sticks as have two or more sides parallel.

1. Required the solidity of a stick of timber that is 10 by 10 in. at one end, and 4 by 4 in. at the other end; and is 32 feet long; measured by taking the mean between the two ends?

Sides $10+4=14\div2=7$ mean side.
Mean side $7\times7\times32=1568\div144=10\frac{8}{9}$ Ans.
Solidity too small by $\frac{8}{9}$ of a foot, (see quest. 1, case 22.)

2. What is the solidity of a stick of timber, which on one side is 15 in. at one end, and 12 the other; and on the other side it is 9 in. at one end, and 6 the other; and is 21 feet in length?

Ends $15+12=27\div\frac{1}{2}=13\frac{1}{2}$ mean side.
Ends $9+6=15\div\frac{1}{2}=7\frac{1}{2}$
Sides $^{2\,7}_{2}\times{}^{1\,5}_{2}\times{}^{2\,1}_{1}\div{}^{1}_{1}\,{}^{4}_{4}=14\frac{4}{5}\frac{1}{7}\frac{1}{6}$ ft. Ans.
and is $1\frac{3}{5}\frac{1}{7}\frac{5}{6}$ ft. smaller than the true answer. (See question 2, case 22.)

NOTE.—This last method of measuring hewn tapering timber, being more convenient than the method of measuring it as a frustum of a cone, it is generally put in practice.

CASE XXIII.

To find the solidity of timber by Gunter's sliding rule allowing ¼ of the circumference to be a side of square timber in the round stick.

RULE 1.—(1) Look first for the length of the stick in feet upon the brass slider, slip the slider to bring the figures expressing the length, to 12 on the girt line below, then look on the girt line for the quarter girt of the stick, and against the quarter girt (on the slider) stands the figures expressing the solidity of the stick.

To find the solidity of a stick of timber, allowing 4·4 of the circumference to be a side of square timber.

RULE 2.—(2) Girt the stick where it would tip easy if it were lying upon another log, annex a cypher to the girt in inches, and divide by 4·4, the quotient is a side of square timber: then proceed as above.

To find the solidity of hewn timber by Gunter's rule.

RULE 3.—(3) Add the four sides together, take ¼ of the sum for the mean side, proceed as above, and the answer will be very near the true solidity.

Questions.—1. What rule have you for finding the solidity of timber by Gunter's rule, allowing ¼ of the circumference to be a side of square timber in the round stick?—2. What is the 2nd Rule, allowing 4·4 of the circumference to be a side of square timber?—3. How do you find the solidity of hewn timber by Gunter's rule?

CASE XXIV.
WOOD AND BARK MEASURE.*

RULE.—(1) Multiply the length of the pile in feet and inches, by the width, and that product again by the height

* Wood and bark are sold by the cord, and 1728 inches make 1 solid foot; 16 solid feet make 1 foot of wood; 8 feet of wood 1 cord; 128 solid feet one cord.

of the pile, the last product is the solidity; which (2) divided by 128 the quotient is cords; if any thing remain, (3) divide it by 16, the quotient is feet of wood; if any thing still remain, (4) divide it by 4, the quotient will be quarters of a foot of wood, &c.

Questions—1. What is the rule for finding the solidity of a pile of *wood* or *bark*?—2. After finding the solidity in feet, how do you reduce it to cords?—3. If any thing remain, what do you do?—4. Should any thing still remain after dividing by 16, what must be done?

EXAMPLES.

1. What is the quantity of wood in a pile, that is 9 feet 6 inches long, 6 feet wide and 4 feet 6 inches high?

ft. ft. ft. ft.

9 6′×6 0′×4 6′=256 6′÷128=2 cords 2 solid ft. Ans.

2. What is the solidity of a pile of wood 22 ft. 9 in. long, 5 ft. 8 in. wide, and 2 feet 6 inches high?

Ans. 2 † cords.

3. What is the solidity of a pile of bark that is 9 feet long, 10 ft. 6 in. high, and 4 ft. 6. in. thick?

Ans. 425¼ sol. ft. or 3 cords 2¼† ft.

CASE XXV.

Having the base, or bottom of a pile of wood, or bark given, to find how high to build, that the pile may contain any quantity of wood, or bark required.

RULE.—(1) Find the solid feet contained in the pile of wood you would build; reduce these feet to solid inches, and use them for a dividend; find the superficial area of the base in inches, and use them for a divisor; divide, and the quotient is the inches in height.

Question.—1. Having the base or bottom of a pile of wood or bark given, how do you find the height to build, that it may contain any quantity of wood or bark required?

EXAMPLES.

1. How high must a pile of wood or bark be to contain 8 ft. or 1 cord; which stands upon a base 48 in. by 48 in.?

1 cord=128 sd. ft. ×1728=221184 sd. in. dividend.

48 in. ×48 in. =2304 area of the base, divisor.

221184÷2304=96 in. or 8 ft. in height, Ans.

2. I demand the height of a pile of wood that will contain 4½ feet, the base or bottom of which is 4 by 5 feet.

Ans. $43\frac{576}{2880}$ in. or 3 ft. $7\frac{1}{5}$ in.

3. I demand the height of a pile of wood, that shall contain 9 feet, that stands upon a base that is 6 by 5 feet.

Ans. $57\frac{2}{4}\frac{5}{3}\frac{9}{2}\frac{2}{10}$ in. or 4 ft. $9\frac{3}{5}$ in.

CASE XXVI.

To find the solidity of a parallelopipedon.

DEFINITION.—A parallelopipedon is (1) a figure having six rectangular sides, every opposite pair of which are equal and parallel.

RULE.—(2) Multiply the length, breadth and thickness together, the product is the solidity.

Questions.—1. What is a Parallelopipedon?—2. How do you find the solidity of such a figure?

EXAMPLES.

1. A farmer would make a large chest 21 ft. long, 5 ft. 6 in. wide, and 3 ft. 9 in. high, the solidity of which is required.

21 ft. 0 in.×5 ft. 6 in.×3 ft. 9 in.=433 ft. 1′6″. Ans.

2. How many solid feet in a cart that is 8 ft. long, 4 ft. 4 in. wide, and 2 ft. 4 in. high? Ans. 80 ft. 10′ 8″.

CASE XXVII.

To find the solidity of any cubical figure.

DEFINITION.—(1) Any figure having six square, or equal sides is called a cube.

RULE.—(2) Multiply one side into another, and this product by the third, the last product is the solidity.

Questions.—1. Describe a cubical figure.—2. How is the solidity of this figure found?

EXAMPLES.

1. What is the solidity of a tea chest that is 3 ft. 6 in. every way?

3 ft. 6′×3 ft. 6′×3 ft. 6′=42 ft. 10′ 6″. Ans.

2. How many solid feet will 60 bales of goods occupy in a store; each bale measuring 4 ft. 3 in. every way?

Ans. 4605 ft. 11′ 3″.

3. How many solid feet are in a square room that is 12 feet long, 12 feet wide and 12 feet high?

Ans. 1728.

4. How many solid feet in a pillar that is 16 feet square?

Ans. 4096.

5. There is a pit dug 8 feet deep, 8 feet long and 8 feet wide; how many solid feet of sand were thrown out of it?
Ans. 512.

GUAGING.

DEFINITION.—Guaging is (1) the art of measuring all kinds of vessels, and determining the quantity of liquor contained in them. Practitioners in the art, generally make their calculations (2) by means of instruments; the instruments used in guaging, are the (3) calipers, Gunter's sliding rule, Gunter's scale, &c. the guaging rod is used (4) to take or find the outs. The vessels used for liquor are various in shapes and names, viz. (5) pipes, hogsheads, barrels, runlets, &c. also, among brewers, coolers, backs, vats, &c. are used.

Solidity of liquid measure, &c.

A gallon of wine occupies a space equal to (6) 231 sol. in.
A gallon of beer occupies a space equal to (7) 282 sol. in.
A gallon in corn measure is equal to (8) 268·8 sol. in.
A bushel in corn measure is equal to (9) 2150·4 sol. in.

Generally speaking there are (10) three sorts of casks, which are judged of according to the curvature of their sides, viz.

1. If the sides of the cask are very curving from end to end, it may be said to represent (11) the middle frustum of a spheroid, (see the figure E F G H, case 13, solids.)

2. If the sides of the cask are more straight it more properly represents (12) the middle frustum of an eliptic spindle, (see figure, case 14, solids.)

3. If the cask is straight from the end to the bung, and of a true taper, (such a cask is rarely found) it represents (13) two frustums of a round cone; the two bases form the middle diameter. (See the frustum of a round cone, case 5, solids, figure first,) which represents half of such a cask.

GENERAL RULE.

(14) Find the solidity of either of the above figures, (or any other figure) in solid inches, and divide the solidity by 231 for wine gallons; by 282 for beer gallons; and by 268·8

for gallons of grain or corn; and by 2150·4 for bushels of corn; and the quotients will be the answers respectively.

Questions.—1. What is Guaging?—2. By what means do practitioners generally make their calculations?—3. What are the instruments used?—4. For what is the guaging rod used?—5. What vessels are most commonly used for liquors?—6. How many solid inches of space does a gallon of wine occupy?—7. How many a gallon of beer?—8. How many a gallon of corn?—9. How many a bushel of corn?—10. Generally speaking how many sorts of casks are there?—11. What does the first represent?—12. What does the second represent?—13. What does the third represent?—14. What is the general rule for finding the content of the before described casks?

CASE I.

To guage a spheroidical cask.

RULE 1.—(1) To twice the square of the bung diameter, add the square of the head diameter, and multiply the sum by the length of the cask, divide the product by 882 for wine, by 1077 for ale, or beer, and by 1026·3 for gallons of corn or grain; and the quotients will be the answers respectively.

RULE 2.—Find the solidity of the cask by the following rule, viz. (2) To the square of the diam. at the end, add twice the square of the middle, or bung diameter; multiply the sum by the length of the cask, and again by ·2618, the last product is the solidity; which divide by 231 for wine; 282 for ale, or beer; and by 268·8 for gallons of corn, or grain: the quotients will be the answers respectively.

Questions.—1. What is the rule for guaging a spheroidical cask?—2. What is the 2nd Rule for finding the content of such a cask?

EXAMPLE.

What is the content of a spheroidical cask, in wine, ale or beer and corn gallons; whose middle or bung diameter B D (see figure, case 13, solids) is equal to 20 inches, and the end diameter E G or F H is equal to 18 inches, and the length of the cask 22 inches?

Solved by rule first.

Bung diam. B D. $20\times20\times2=800$ twice the square.
Head diam. E G or F H $18\times18=169$ added.

sum $969\times22=21318$ pro.

Pro. $21318\div882=24\frac{150}{882}$ gal. of wine, Ans.

$21318\div1077=19\frac{945}{1077}$ gal. of ale or beer, Ans.

$21318\div1026\cdot3=20\frac{7929}{10263}$ gal of corn, &c. Ans.

The same question by rule second.

Diam. E G or F H 13 × 13=169 square of E G or F H.
Bung diam. B D 20×20×2=800 twice square B D add.

969 sum

Sum 969 × 22 length=21318 × 2618=5581·0524 sol.
Solidity 5581·0524÷231=24·16† gal. of wine, Ans.
5581·0254÷282=19·79† gal. of ale, &c. Ans.
5581·0524÷268·8=20·76† gal. of corn, &c. Ans.

NOTE.—Dimensions are always taken in inches; due allowance must be made for the thickness of the heads, &c.

CASE II.

To guage a cask representing the middle frustum of an elliptic spindle.

RULE. 1—(1) To twice the square of the bung diameter, add the square of the head diameter and reserve the sum; find the difference between the head and bung diameter, square the difference, and subtract $\frac{2}{3}$ of the square of the difference from the reserved sum; multiply the remainder by the length of the cask; divide the product by 882 for wine; by 1077 for ale or beer; and by 1026·3 for corn, &c; the quotient will be the answers *in gallons* respectively.

RULE 2.—Find the solidity in solid inches by the following rule, viz. (2) To the square of twice a diameter taken half way between the head and bung, (as at o P) add the squares of the head and bung diameters; multiply the sum by the length of the cask, and again by 1309 for the solidity; divide the solidity by 231 for wine; and 282 for ale, or beer; and by 268·8 for corn, &c.; the quotients will be the answers in gallons respectively.

Questions.—1. What is the 1st rule for guaging a cask representing the middle frustum of an elliptic spindle?—2. What is the 2nd rule?

EXAMPLE.

What is the content of a cask representing the middle frustum of an elliptic spindle, (see figure E F, O P, C D, and G H, case 14, solids) in wine, ale, and corn gallons; whose length is 38; bung diameter C D 22; and end diameter E F, or G H. 16: and a diameter at O P 20·5 inches?

Solved by rule first.

Diam. C D $22\times22=484\times2=968$
Diam. E F or G H $16\times16=256$

reserved sum 1224
Diam. $22-16=6\times6=36\div\frac{2}{5}=14\cdot4$ sub.

remainder $1209\cdot6$
rem. $1209\cdot6\times38=45964\cdot8$ product.
pro. $45964\cdot8\div882$ $=52\cdot1\dagger$ gal. wine, Ans.
$45964\cdot8\div1077$ $=42\cdot6\dagger$ gal. ale, &c. Ans.
$45964\cdot8\div1026\cdot3=44\cdot7\dagger$ gal. corn, &c. Ans.

The same question by rule second.

Diam. at O P. $20\cdot5\times2=41\times41=1681$ square of twice O P
Diam. C D 22×22 $=484$ square of C D
Diam. E F or G H $16\times16=256$ square of E F or GH

2421 sum.
the sum $2421\times38\times\cdot1309=1$ $2042\cdot5382$ solidity.
Solidity $12042\cdot5382\div231=52\cdot1\dagger$ gal. wine, Ans.
$12042\cdot5382\div282=42\cdot7\dagger$ gal. of ale, &c. Ans.
$12042\cdot5382\div268\cdot8=44\cdot8\dagger$ gal. of corn, &c. Ans.

CASE III.

To guage a cask representing two frustums of a round cone, the bases being joined together to form the bung diameter, (*See figure* 1, *Case* 5, *Solids.*)

RULE 1.—(1) Add the two diameters together, square the sum and multiply the square by 3, and reserve the product; find the difference of the diameters; square the difference, and add the square of the difference to the reserved product, and multiply the sum by the length, and the product again by $\cdot00023\frac{1}{2}$ for ale, &c. by $\cdot00029\frac{1}{4}$ for wine.

RULE 2.—Find the solidity of half the cask by the following rule, (half of such a cask would represent a churn, or mash tub, or basket,) viz. (2) Find the area, of the two ends,* supposing the cask to be cut in two, add the areas together and reserve the sum; multiply the areas together, extract the square root of the product, and add the root to the reserved sum; multiply the sum by $\frac{1}{2}$ the length and $\frac{1}{3}$

* To find the area of the ends, (3) multiply the diameter by the circumference, and $\frac{1}{4}$ of the product is the area.

the product is the solidity of half the cask, which must be doubled to make the solidity: divide the solidity by 231 for wine; 282 for ale; by 268·8 for corn gals. &c.

Questions.—1. What is the first rule for guaging a cask representing two frustums of a round cone, the bases being joined together to form the bung diameter?—2. What is the second rule?—3. *How is* the area of the ends found?

EXAMPLE.

What is the content of a cask representing two frustums of a cone, (see frustum A B C D, figure first, case 5, solids) diameter A B 31, and diameter, C D 44; length 55?

Solved by rule first.

res'd pro.

Diam A B & C D 31+44=75×75=5625×3=16875

Diam. C D & A B 44—31=13×13=169 add.

Sum 17044

Sum 17044×55×·00023½=217·48144 ale gal. Ans.

17044×55×·00028⅓=265·60233 wine gal. Ans.

The same question by rule second.

As 7 : 22 : : 44 diam. : 138·3† circum.

7 : 22 : : 31 diam. : 97·4† circum.

Diam. 44×by cir. 138·3÷4=1521·3 area largest end.

Diam. 31×by cir. 97·4÷4= 754·8† area of small. end.

Areas. 2276·1 reserved sum.

1521·3×754·8=√1148277·24=1071·5 root added.

3347.6 sum

Sum 3347·6×half length 27·5÷4=30686·33 solidity of half the cask.

30686·33×2=61372·66 solidity of the cask.

solidity 61372·66÷282=217·6† ale gal. &c Ans.

61372·66÷231=265·6† wine gal. Ans.

GUAGING, SECOND METHOD.

Sometimes the following method is used in guaging.

RULE.—(1) Measure the diameter of the cask at both ends, and take half the sum for the head diameter; measure the diameter at the bung, or in the middle of the cask; (dimensions must be taken upon the inside, or else allowance must be made for the thickness of the staves, &c.;) measure also the length of the cask; find the difference be-

tween the head and bung diameter; multiply it by ·62, and add the product to the head diameter, the sum is the mean diameter of the cask; (and reduces it to a cylinder.) Square the mean diameter and multiply the square by the length of the cask, and divide by 294·12 for wine; by 359·05 for ale, &c.; by 342·25 for corn gallons; and by 2738 for bushels of corn.*

Question.—1. What is the general rule to be used in the second method of Guaging?

EXAMPLES.

1. What is the content of a cask in form of the figure in case 13th, solids, whose diameter E G or F H equal 13, and diameter B D 20, and length 22 inches?

Diam. 20—13=7 diff. ×·70=4·90+13=17·9 mean diam.
diam. 57·9×17·9×22=7049·02 product.
Product 7049·02÷294·12=24† gal. wine, Ans.
7049·02÷359·05=19† gal. ale, &c. Ans.
7049·02÷342·25=20† gal. of corn, Ans.

If the cask is more straight, the number ·67 ought to be used.

2. What is the content of a cask representing figure E F C B G H, case 14th solids; diam. E F or G H equal 16, diam. C D 22; length 38 inches?

Diam. 22—16=6 dif. × 67=4·02+16=20·02 mean diam.
Mean diam. 20·02×20·02=400·8004×38 len.=15230·4152 pro.
Pro. 15230·4152÷291·12=51† gal. wine, Ans.
15230·4152÷359·05=42† gal. ale, &c. Ans.
15230·4152÷342·25=44† gal. of corn, Ans.

3. What is the content of a cask, formed by joining two frustums of a round cone together, (the frustum A B C D figure first, case 5th, in solids, represents half of such a cask) diam. 44 and 31, and length 55 inches?

* These divisors arise by dividing 231, 282 and 268·8 by ·7854; if the square of the mean diameter were multiplied by the length and again by ·7854, the last product would be the solidity of the cask; (see Cylinder, case 4th solids) the solidity being divided by 231 for wine; 282 for ale; by 268·8 for gallons of corn, &c. the quotients would be the answers in gallons, respectively; although the number ·62 is generally used by guagers, yet it ought to be varied according to the curvature of the cask; if the cask is very curving like figure, case 13th, solids, it would require that the difference between the diameters be multiplied by ·69 or ·7; if the cask is more straight the number used should be less.

NOTE.—A cask of this form requires that the difference between the diameters be multiplied by ·515, for 52 produces too much.

Diam. 44—31=13 dif.×·515=6·695×31 diam.=37·695 mean diam.

Mean diam. 37·695×37·695×55=78150·216375 pro.

Pro. 78150·216375÷294·12=265·7† gal. wine, Ans.
78150·216375÷359·05=217·6† gal. ale, &c. Ans.

NOTE.—It is evident from the three last examples, that the number 62 would not exactly apply in either case: the dimensions of the three last examples are the same as the dimensions of the three first examples, by the first method; and the numbers which I have made use of produce the same answers as were produced by the first method. If the cask is a little straighter than the elliptic spindle, in case 14 solids, the number ·62 will exactly apply: this number should always be varied according to the shape of the cask, and at the judgment of the guager.

4. What is the content of a spheroidical cask whose length is 20; diameters 16 and 12 inches?

Ans. 14·8† wine, 12·2† ale, &c.

In this example the number ·7 is used.

5. What is the content of a cask representing an elliptic spindle, length 40; diameters 32 and 24?

Ans 117·2† wine, 96·0† ale, &c.

In this example number ·67 is used.

6. What is the content of a cask, representing two frustums of a cone, diam. 24 and 32; length 40?

Ans. 107·5† wine, 88·09† ale, &c.

In this example the number ·515 is used.

To guage with calipers, or Gunter's scale.

(1) On the calipers there is a brass pin fixed at the points; 17·15 and at 18·95;* these are called guage points; 17·15 is used for guaging wine, and 18·95 is used for ale, or beer, &c.

RULE.—For a cask representing the middle frustum of a spheroid, (see figure, case 13, solids) (^{2}) extend from 1 to ·7; and with the same extent, set one foot of the dividers in the number expressing the difference between the bung and head diameter, and they will reach to a number, towards the left hand, which, when added to the head diameter will give the mean diameter of the cask; then set one foot of the dividers in the guage point, and extend to the mean diameter; and the same way set off twice that extent from the length of the cask, and they will reach to a number expressing the content of the cask.

* These numbers are the square roots of the numbers 291·12 and 859·05 respectively.

NOTE 1.—The same operation may be performed on the line of numbers on Gunter's scale.

NOTE 2.—(3) If wine gallons are required, use the guage point 17·15; if ale gallons are required, use the guage point 18·95; and if corn gallons are required, you may use the point 18·5 respectively.

NOTE 3.—(4) If the cask represents the middle frustum of an elliptic spindle, (see figure, case 14, solids) extend from 1 to ·67, or if a little more straight, extend from 1 to ·62; and for a cask formed by two frustums of a cone, extend from 1 to ·51 or ·52.

Questions.—1. Describe the calipers or compasses used for Guaging?—2. How do you find the content of a cask by this instrument, or Gunter's scale?—3. Give the *guage points* to be used for the different measures required.—4. Give the points to extend from and to according to the figure or representation of the cask.

EXAMPLES.

1. What is the content of a cask of rum, or wine, whose length is 59·3 in.; and diameters 34·5, and 30·7; difference of diameters 3·8?

Operation.

Extend from 1 to ·62; and that extent reaches from 3·8, (the difference of the diameters) towards the left hand to 2·4 nearly, which add to the head diameter; 30·7+2·4=33·1 mean diam.; then extend from the guage point 17·15 to 33·1 (the mean diameter,) and set off twice that extent from 59·3 (the length of the cask,) and they will reach to 220·8 nearly, which is the content of the cask.

2. What is the content of a cask of wine whose length is 38; diam. 22, and 16 in.: difference of diameters 6?

Ans. 52† wine, 42† ale, &c.

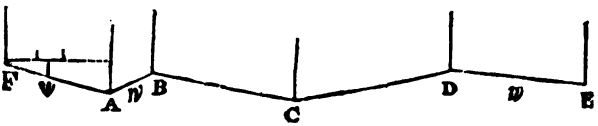
Operation.

Extend from 1 to ·67; and that extent will reach from 6, (the difference) to 3·7 nearly: which being added to the head diameter 16, produces 19·7 for the mean diameter: then extend from the wine guage point to 19·7, (the mean diameter) and set off twice that extent from 38, the length; and the dividers will rest on 52 the answer.

NOTE.—Guagers make use of the guaging rod to take the dimensions of the cask; and there are also tables on it by means of which the outs are taken. After the preceding rules are well committed to memory, the young practitioner would soon obtain a complete knowledge of guaging, by a little practice with the instruments.

WATER LEVELLING

To find whether water can be made to run from one place to another, or to find how much higher one place is than another.



F is the fountain, and E the place assigned to carry the water to.

NOTE.—To perform this business you must be provided with two staves divided into inches and tenths, which may be of any convenient length, perhaps 4 or 6 feet; and also a water level which may be made easily at any joiner's shop; the level may be of any convenient length, dug out hollowing, so as to hold water; with two sights at the top to make the observation through; having provided the staves and water level, go to the fountain with two assistants and observe the following rule.

RULE.—Order the first assistant to the fountain with one staff placed perpendicularly: and the second assistant to any convenient place as at A, with his staff perpendicularly; then place the water level in the middle between them at *w*, and looking through the sights order the first assistant to move a piece of white paper up and down the staff till you can see it through the sights; then order him to notice the distance it rests from the ground; and going to the other end of the water level, order the second assistant to do likewise; and after he has marked down the distance the papers rest from the ground, order the first assistant to take the place of the second, and the second to take a new stand as at B; place the water level between them, make observations as before, and order the assistants to mark down the distances the paper rests from the ground, and proceed to take another station and make new observations till you have arrived to the place assigned; then order the assistant's to cast up their notes, and as much as the second assistant's notes exceed the first, so much does the ground descend; and so much as the second assistant's notes are

less than the first, so much does the ground rise. In the preceding example the assistant's notes are as follows:

First assistant's notes.

	ft.	*in.*
First station	0	7
Second do.	1	0
Third do.	0	2
Fourth do.	1	2
Fifth do.		4
ft.	3	3 in.

Second assistant's notes.

	ft.	*in.*
First station	1	0
Second do.	0	6
Third do.	1	6
Fourth do.	0	2
Fifth do.	0	7
ft.	3	9 in.

Second assistant's notes exceed the first by 6 inches; of course the ground is 6 inches lower at E, than at E, and water will run.

MISCELLANEOUS QUESTIONS.

1. At a certain election 375 persons voted for 2 candidates, and the candidate chosen had a majority of 91; how many voted for each? Ans. 233 and 142.

2. Suppose a man to step 30 inches at a time, and to go 4 miles an hour, how many times does he step in a minute?
Ans. 104$\frac{1}{2}$.

3. The divisor is 43967, the quotient 2737226, and the remainder 27672; what is the dividend?
Ans. 120347643214.

4. After paying away $\frac{1}{4}$ and$\frac{1}{2}$ of my money, I had 66 guineas left in my purse; what was in it at first?
Ans. 120.

5. A water-tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes; the tap discharges, at a medium, 40 gallons in 31 minutes; now supposing these both to be carelessly left open, and the water to be turned on at 2 o'clock in the morning; a servant at 5, finding the water running, shuts the tap, and is solicitous to know in what time the tub will be filled after this accident, in case the water continues to flow from the main.
Ans. the tub will be full at 3m. 48$\frac{1}{31}\frac{4}{7}$ sec. after 6.

6. If one pound, ten, and 40 groats,
Will buy a load of hay;
How many pounds with 19 crowns
For 20 loads will pay? Ans. £38 11s. 8d.

7. A man being asked how many sheep he had in his drove, said, if he had as many more, half as many more, and 7 sheep and an half, I should have 20: how many had he? Ans. 5.

8. I want a tub made, of equal bigness from top to bottom, that will hold 10 bushels, the diameter of which is limited to 28 inches, how high must it be made?
Ans. 36·65† inches.
(*Call a bushel dry measure* 2150$\frac{1}{2}$ *solid or cubic inches.*)

9. An elm plank is 14 feet 3 inches long; what distance from the edge must a line be struck to take off a yard square?

Ans. $7\frac{11}{13}$ inches.

10. A man, having a son at sea, made, upon his death bed, a will, in which he provided, that if the son returned, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; but if the son should not return, he appointed the mother $\frac{1}{3}$, and the remainder to her daughter, provided she should marry. The son was lost, and the daughter married, by which act the mother lost in equity $2400 more than if the son had returned and the girl had not married. What would have been her dower if the son had returned, and the daughter remained single?

11. A lent B a piece of cedar board 6 feet square and has received two pieces each 3 feet square; who is the person indebted, and how much? Ans B owes A 18 feet.

12. The wall of a town is 17 ft. high, which is surrounded by a ditch of 20 feet in breadth; I demand the length of a ladder that shall reach from the outside of the ditch to the top of the wall? Ans. 26·2 feet nearly.

13. How many feet of boards will it take to cover a barn which is 76 ft. long, 32 ft. wide 14 ft. post, and 20 ft. spar; and how many shingles will it require to lay the roof, admitting 6 shingles to lay a superficial foot?

Ans. 6600 ft. boards and 19152 shingles.

NOTE.—The roof boards are allowed to be 21 feet long.

14. What is the difference and what the sum of six dozen dozen, and half a dozen dozen?

Ans. { The sum is 936
{ The difference 792.

15. A owed B $317,19, for which he gave his note on interest, bearing date July 12th, 1797. On the back of the note are these several endorsements, viz.

Oct. 17th, 1797, Received in cash $61,10.

March 20th, 1798, Received 17 Cwt. beef, at $4,33 per cwt.

Jan. 1st, 1800, Received in cash $84.

What was there due from A to B of principal and interest, Sept. 18th, 1801? Ans. $144,363.

16. A traveller leaves Exeter at 8 o'clock on Monday morning and travels at the rate of 3 miles an hour towards London, another traveller sets out from London at 4 o'-

clock the same evening, and walks for Exeter at the rate of 4 miles an hour, both walking without intermission; now supposing the distance from one place to the other to be 130 miles, whereabouts on the road will they meet?

Ans. $69\frac{1}{7}$ miles from Exeter.

17. What is the length of a road, which being 33 feet wide contains an acre? Ans. 80 *rods in length.*

18. A B & C, bought a drove of sheep in company; A paid £14 5s.; B, £13 10s. and C £11 5s. They agreed to dispose of them at the market; that each man should take 18s. as pay for his time, &c. and that the remainder should be divided in proportion to their several stocks: at the close of the sale they found themselves possessed of £46 5s.; what was each man's gain exclusive of the pay for his time, &c.

Ans. { A's gain £1 13s. 3d
B's " £1 11s. 6d
C's " £1 6s. 3d

Whole gain, Proof. £4 11s. 0d

19. A man lays out 80 cents for apples and pears, buying his apples at 4 and his pears at 5 for a cent, and afterwards sold $\frac{1}{4}$ of his apples and $\frac{1}{3}$ of his pears for 18 cents, which was the prime cost; I demand the number he bought of each?

Ans. 72 apples, 60 pears.

20. Four men, A, B, C, & D, found a purse of money containing 12 dollars, they agree that A shall have one third, B one fourth, C one sixth and D one eighth of it, what must each man have according to this agreement?

Ans. A's share, $\$4,571\frac{3}{7}$. B's share, $\$3,428\frac{4}{7}$.
C's share, $\$2,285\frac{5}{7}$. D's share, $\$1,714\frac{2}{7}$.

21. What is the present worth of 482 dollars, payable 4 years hence, discounting at the rate of 6 per cent?

An. $388,709.

22. I have owing to me as follows: viz. $18,73 in 8 mo.: $46,00 in 5 months; and 104,84 in 3 months; what is the mean time for the payment of the whole?

Ans. 4 *months* 2 *days.*

23. A began trade June 1, with 40 dollars, and took in B as a partner, Sept. 8, following, with 120 dollars; on Dec. 24, A put in 190 dollars more, and continued the whole in trade till May 5, following, when their whole gain

was found to be 32 dollars; what is each partner's share?
Ans. *A's share* $47,065 + *B's share* $34,934 +

24. What fraction is that, to the numerator of which, if 1 be added the value will be $\frac{1}{3}$; but if 1 be added to the denominator, its value will be $\frac{1}{4}$? Ans. $\frac{4}{11}$

25. Sold a piece of cloth for £24, and gained as much per cent. as the cloth cost me; what was the price of it?
Ans. 20.

26. There is an island 50 miles in circumference, and three men start together to travel the same way about it; A goes 7 miles per day, B 8, and C 9; when will they all come together again, and how far will they travel?
Ans. 50 *days.* A 350 *miles.* B 400, *and* C 450.

27. A said to B & C, give me $\frac{1}{2}$ of your money and I shall have £100, B said to A and C, give me $\frac{1}{3}$ of yours and I shall have £100, C said to A and B, give me $\frac{1}{4}$ of your money and I shall have £100, How much money had each man? Ans. A had £$29\frac{7}{11}$, B had £$64\frac{2}{11}$ C had £$76\frac{4}{11}$.

28. A man driving his geese to market was met by another, who said, Good morrow master, with your hundred geese; says he, I have not an hundred, but if I had half as many as I now have, and two geese and a half beside the number I now have already, I should have an hundred. How many had he? Ans 65 geese.

29. A man had an hundred dollars with which he bought 100 animals, consisting of oxen, sheep and geese; for oxen he gave $10 a piece, for sheep $1, and for geese 1 shilling: How many of each did he buy?

Ans. 54 Geese= 9 dolls,
5 Oxen =50 "
41 Sheep=41 "

$100 Proof.

30. A hare starts 12 rods before a hound; but is not perceived by him until she has been up 45 seconds; she scuds away at the rate of ten miles an hour, and the dog on view, makes after at the rate of 16 miles an hour. How long will the hound be in overtaking the hare, and what distance will he run? Ans. 97½ *seconds, he will run* 2288 *feet.*

31. If by selling goods at 2s. 3d. per lb. I clear cent per cent, what do I clear per cent, by selling them at 9s per cwt? Ans. 50 per cent.

32. A general disposing his army into a square battalion, found he had 231 over and above; but increasing each side with one soldier, he wanted 44 to fill up the square; Of how many men did his army consist? Ans. 19000.

33. A can do a piece of work in 8 weeks, B can do thrice as much in 8 weeks, and C 5 times as much in 12 weeks; in what time can they finish it jointly?

Ans. 5 days 4 hours.

34. A military officer drew up his soldiers in rank and file, having the number in rank and file equal; on being reinforced with three times his first number of men, he placed them all in the same form, and then the number in rank and file was just double what it was at first; he was again reinforced with three times his first number of men, and after placing the whole in the same form as at first, his number in rank and file was 40 men each; How many men had he at first? Ans. 100.

35. If a steam engine can, working alone, do a certain amount of work in 1 hour, a water-mill in 3 hours, a wind-mill in 5 hrs. and a horse-power mill in 7 hrs.; what time will it require for the whole, working together at the same time, to do three times the amount of work? Ans. 1 ho. 47 m. $23\frac{2}{11}$ sec.

36. A and B purchased 20 acres and 72 rods of land at $25·57 cts. per acre. By agreement A was to have 6 acres and 12 rods of the land, and B to have the remainder, and to pay $4 per acre more than A. How much did each man's land cost him per acre? Ans. A's cost $22·75·8m.
B's " $26·75·8m.

37. How many yards of stuff that is 3 quarters of a yard wide will line a cloak, in which there are 4 yards of cloth that was 7 quarters wide? Ans. $9\frac{3}{7}$ yds.

38. A hare is 50 leaps before a grey hound, and takes 4 leaps to the grey hound's 3; but 2 of the grey hound's leaps are as much as 3 of the hare's; how many leaps must the hound take to catch the hare? Ans. 300.

39. How many 3 inch cubes may be cut out of a 12 inch cube? Ans. 64.

40. If the area of a triangle be 160, what is the side of a square equal in area thereto? Ans. 12·649.

41. A B C & D companied, and gained a sum of money, of which A B & C took $500; B C & D $400; C D & A

$880 and D A & B $360; what was the gain of each?

Ans. A's gain $80.
B's " $100.
C's " $120
D's " $180.

42. A person looking on his watch, was asked the time of day, said, that it was between 3 and 4, and that the hour and minute hands were then together; what was the time?

Ans. $16\frac{4}{11}$ minutes past 3.

43. What sum of money will produce as much interest in $3\frac{1}{4}$ years, as $210·15 cts. can produce in 5 years and 5 mo.?

Ans. $350·25 cts.

44. What is the gross weight of a hhd. of tobacco, weighing net 11 cwt. 1 qr.; tare 14 lb. per cwt.?

Ans. 12 cwt. 3 qr. 12 lb.

45. A person being asked in the afternoon what o'clock it was, answered, that the time past from noon was equal to $\frac{1}{3}$ of the time to midnight. Required the exact time?

Ans. 5 hours 20 min.

46. What part is $28\frac{1}{12}$ of $33\frac{1}{11}$? Ans. $\frac{7}{8}$.

47. Which is the greater fraction $\frac{9}{11}$ or $\frac{9}{20}$, and by how much? Ans. $\frac{9}{11}$ is greater by $\frac{1}{220}$.

48. B & C purchased 1200 acres of land at $1 per acre, each paying $600. Some time after, C on viewing it, offers to take a certain square piece at $1·75 per acre to the amount of his advance, to which B consents. How many acres will each have, what is the length of each side of C's lot, and what does B's part cost him per acre?

Ans. C has 342 acres 3 roods $17\frac{1}{4}$ rds.
B has 857 acres 0 roods $22\frac{3}{4}$ rds.
B's land is ·70 cts. per acre.
Side of C's=234 rods 3 ft. $6\frac{3}{4}$ in.

49. If by selling goods at $2½ per cwt. I gain 20 per cent., what do I gain or lose per cent. by selling at $2¼ per cwt?

Ans $8 per cent. gain.

50. Sold a piece of cloth containing 1000 Flemish Ells for 850 guineas, and gained upon every yard $\frac{1}{8}$ of the prime cost of an English Ell: what did the whole piece stand me in? Ans. £771·17s. $10\frac{2}{37}$d.

51. Three men are employed to perform a piece of work for £2 10s.; A & B are supposed to do $\frac{3}{11}$ of the work,

A & C $\frac{4}{13}$, and B & C $\frac{8}{17}$ of it; they are to be paid proportionally; can you divide it as it should be?

Ans. A's proportion is 19s. 8d. $1\frac{4}{13}$qr.
B's " " 9s. 2d. $2\frac{2}{17}$qr.
C's " " £1 1s. 0d. $3\frac{1}{13}$qr.

Proof=£2 10s. 0d. 0qr.

52. A certain usurer lent £90 for 12 months, and received principal and interest £95 8s. I demand at what rate per cent he received interest. Ans. 6 *per cent.*

53 If a gentleman have an estate of £1000 per annum, how much may he spend per day to lay up three score guineas at the year's end? Ans. £2 10s. 2d. $1\frac{1}{73}$q.

54. A B & C have among them 135 guineas; A's+B's are to B's+C's, as 5 to 7, and C's—B's to C's+B's as 1 to 7; How many had each? Ans. A had 30
B " 45
C " 60

Proof 135 Sum.

55. Two ships A and B sailed from a certain port at the same time; A sailed north 8 miles an hour, and B east 6 miles an hour; What was their distance at the end of one hour? Ans. 10 *miles*

56. After an old man's death, in gold was found
Left to his family, eight thousand pounds,
To be bestowed as his last will directed
Which did provide that none should be neglected:
For to each son, there being in number five
Three times each daughter's portion he did give
The daughters each were also to receive,
Double the sum he to their mother gave;
His daughters, all in number, were just four,
Their gold in weight, eight times their mother's store:
Now that this Will may justly be fulfilled,
What must the widow have and what each child?

Ans. Mother's share £205 : 2s. $6\frac{6}{17}$d.
Daughter's " £410 : 5s. $1\frac{7}{17}$d.
Son's " £1230 : 15s. $4\frac{5}{17}$d.

57. Bought 30 hogsheads of molasses at $600; paid in duties $20,66; for freight $40,78; for porterage $6,05, and for insurance, $30,84; If I sell it at $26 per hogshead, how much shall I gain per cent? Ans. $11,695.

58. A Stationer sold quills at 11s. per thousand, by which he cleared $\frac{1}{8}$ of his money; but growing scarce, he increased them to 13s. 6d. a thousand; what did he clear per cent by the latter price? Ans. £96 7s. 8$\frac{1}{2}$d.

59. If I sell 500 deals at 15d. a piece, and lose £9 per cent., what do I lose in the whole quantity?
Ans. £2 16s. 3d.

60. If I buy 1000 Ells Flemish of linen for £90, what may I sell it per Ell in London, to gain £10 in the whole?
Ans. 3s. 4d.

61. How many wine gallons in a cask, whose bung diameter measures 27 inches, head diameter 21 inches, and length 30 inches? Ans. 63 gals. 3 qts.

62. A and B companied; A put in £45, and took $\frac{3}{5}$ of the gain; What did B put in? Ans. £30.

63. A fellow said that when he counted his nuts two by two, three by three, four by four, five by five, and six by six, there was still an odd one; but when he counted them seven by seven they came out even; How many had he?
Ans. 721.

64. A and B cleared by an adventure at sea 45 guineas, which was £35 *per cent.* upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B; what money did each adventure?.
Ans. A. £104 4s. 2$\frac{10}{19}$d. B £75 15s. 9$\frac{9}{19}$d.

65. C D & E company and put in together $3860, C's money was in 3 months; D's money was in 5 months; and E's was in 7 months; they gained $234, which was so divided, that $\frac{1}{2}$ of C's gain was equal to $\frac{1}{3}$ of D's, and $\frac{1}{4}$ of D's was equal to $\frac{1}{7}$ of E's; what did each gain and put in?

Ans. C. gained	$52	and put in	$1400
D. "	78	"	1260
E. "	104	"	1200
Sum gained,	$234.	Am't put in,	$3860. proof.

BOOK-KEEPING.

A

SHORT SYSTEM OF

BOOK-KEEPING.

BY SINGLE ENTRY.

DESIGNED

FOR THE USE OF

FARMERS, RETAILERS, AND MECHANICKS.

S2

ADVERTISEMENT.

ALTHOUGH systems of Book-Keeping are numerous, yet it is seldom that any of them get into any of our country schools; and it is very common to find young men, sons of reputable Farmers, Mechanicks, and Tradesmen, who have never seen a system of Book-Keeping; and when they become of age and enter upon business for themselves, they have to contrive some system for their own use. Having frequently seen instances of this kind, I thought a short practical system of Book-Keeping, suitable for Farmers, Mechanicks, and Retailers, would, by being thrown into the hands of boys in our country schools, conduce greatly in removing the ignorance of that useful science, which so generally prevails. THE AUTHOR.

Merchants have various methods of keeping their Books, and many of them keep various Books, such as the Blot-Book, Waste-Book, Day-Book, Ledger, Bill-Book, Sales-Book, Cash-Book, Invoice-Book, &c.; but as this system is designed for Farmers, Mechanicks, and Traders, I shall treat of two books only, viz. the Day-Book and Ledger, these being all that are necessary for that part of the community for which this work is intended.

Of the Day-Book.

Begin your Day-Book with an account or invoice of all your property; charge yourself with all the estate you are in possession of, either real or personal, and credit the same account with all the debts you owe, whether on note or account, and the difference between the debtor and credit side of this account will shew the situation or value of your estate when you commenced business.

When you sell any thing to any person on credit, enter the purchaser *Debtor* in the Day-Book. To the articles sold, naming the month, year, and day of the month; the quantity and price of the article or articles sold and to whom delivered should be particularly noticed; and if any particular time or any particular manner of payment is

agreed upon, it is best to name the circumstances with the charge: if you buy goods of any person enter the person of whom you purchase *Creditor.* By the articles purchased, naming every circumstance agreed upon, time of payment, &c.

Keep an account of all the notes or other instruments you sign, whereby you consider yourself liable to pay; that you may know all the demands against you; and your stock of goods on hand, your notes and balances of your accounts taken at any time when you please, will enable you to ascertain whether you are gaining or loosing by your business.

The following examples will illustrate the process, and give the scholar an idea of commencing business and from time to time to ascertain whether his business is profitable or not.

Abel Flint, a farmer, enters on business in circumstances as follows:

	Abel Flint, Dr.		*Dols. Cts.*
A. D. 1817. Dec. 1.	To sundry goods and chattels belonging to him the first day of December, A. D. 1817, valued as follows, viz.		
	100 acres of land,	$1000	
	4 oxen,	100	
	1 pair wheels, cart, &c.	50	
	1 plow and 1 harrow,	15	
	20 sheep,	40	
	1 horse,	40	
	90 bushels of corn and grain,	90	
	Pork, beef, beans, hay, &c.	180	
	Stephen Goodhue owes me	90	
	Stephen Beedle owes me	150	
	Nathan Hoag owes me	650	2405·00
	Contra,	**Cr.**	
	By sundry debts due from me to sundry persons, December 1, 1817, viz.		
	Due to Peter Wilberforce towards my farm,	$650	
	Due to Seth Wright,	80	
	Due to Nathan Lookout,	122	852·00
			1553·00
	Neat of my estate December 1, 1817, after deducting all demands against it, including interest, &c.	$1553·00	

A. D. 1818. March 20.	**T. Ayer, *of Chester*, Dr.**			*Dols. Cts.*
*	To two bushels corn, at 7s. 6d.		$2·50	
	" six bushels potatoes, 1s. 6d.		1·50	
	" fifteen lbs. cheese	9d.	1·87½	5·87½
	Payable in labor at 50 cts. per day, on demand.			
	David Briant, *of Exeter*, Dr.			
" 23.				
*	To 20 lbs. of flax, at 16 cts.		3·20	3·20
	Enoch Calhound, *of Lee*, Dr.			
" 26.	To 20 lbs. flax, at 16 cts.		$3·20	
*	" 20 lbs. lambs wool 30 cts.		6·00	
	" 4 bushels corn 1·25 "		5·00	
	" 10 bushel potatoes, 25 "		2·50	
				16·70
	T. Ayer, *of Chester*, Cr.			
" 29.	By 12 days labor, at 50 cts.		$6·00	6·00
*	**Contra,**		**Dr.**	
	To 4 bushels corn, at 1·25		$5·00	
*	" 1 and half bushels white beans, at 1·50		2·25	7·25
	Enoch Calhound, *of Lee*, Dr.			
" 30				
*	To 4 bushels of corn, at 1·25		$5·00	5·00
	Tho. Dwight, *of Chester*, Cr.			
A. D. 1818. June 9.	By seven thousands of shingles, at $2 per thousand, payable in			
*	salt at 75 cents per bushel, delivered at Exeter the third day of August next.			14·00

A. D. 1818.			$ cts.
July 30.	Enoch Calhound, *of Lee*, Dr.		
	To two barrels cider, at $2 50,		5 00
" 31.	Thomas Ingals, *of Exeter*, Dr.		
	To one bushel corn, at 1 25,	$1 25	
	" four bush. potatoes, 25,	1 00	2 25
	August, A. D. 1818.		
A. D. 1818.	Thomas Ingals, of Exeter, Dr.		
August 1.	To one pig, 6 weeks old, at 1 50		1 50
	Trustum Ayer, *of Chester*, Dr.		
" 20.	To one barrel cider, at 2 50	$2 50	
	" 1 bushel corn, 1 25	1 25	
	" ½ do. white beans, 2 50	1 25	5 00
" 29.	Enoch Calhound, *of Lee*, Dr.		
	To pasturing a horse six weeks, $6		6 00
	David Briant, *of Exeter*, Dr.		
" 29.	To my horse and chaise to Portsmouth,		1 25
	Trustum Ayer, *of Chester*, Dr.		
" 30.	To myself and four oxen, to haul salt from Exeter to Chester, two days, at $2 per day, - - - - -		4 00
	Enoch Calhound, *of Lee*, Cr.		
" 31.	By cash received by the hand of Nathan Briand, - - - - -		15 00

		$ cts.
	Thomas Ingals, of Exeter, Dr.	
A. D. 1818. Aug. 31.	To 1 bushel of corn, at 1 25, $1 25	
	" 1 do. rye, 1 50, 1 50	
	" 2 do. potatoes, 25, 50	
		3 25
	Trustum Ayer, of Chester, Dr.	
Dec. 10.	To six gallons vinegar, at 33 cts.	1 98
	Thomas Ingals, of Exeter, Dr.	
" 10.	To 1 pr. men's shoes, at 1 75, $ 1 75	
	" 1 peck white beans, 1 48, 37	
	" 1 bushel rye, 1 50, 1 50	
	" 3 do. corn, 1 25, 3 75	
		7 37
	David Briant, of Exeter, Dr.	
" 20.	To 20 lbs. butter, at 18 cts. $3 60	
	" 30 lbs. cheese, 10 3 00	
	" 2 barrels cider, 2 50 5 00	
		11 60
	Thomas Dwight, of Chester, Dr.	
" 31.	To $18\frac{2}{3}$ bushels salt, in full payment for seven thousand shingles, purchased of him, June 9, 1818, at 75 cts.	14 00
	Thomas Ingals, of Exeter, Dr.	
" 31.	To twelve days myself and four oxen, at $2 00, - - - -	24 00

OF THE LEDGER.

The Ledger collects the dispersed accounts of the Day Book, and shows each man's account on a page; the debt on the left hand, and the credit on the right.

Open an account in the Ledger for every person whose name is mentioned in the Day-Book, head the account with the person's name, in a large round hand; then on the left hand page make him Dr. to the several sums you find charged to him in the Day-Book; and on the right hand page give him Cr. by the amount of all you find by the Day-Book you have purchased of him; thus each man's account will be exhibited to view on a page in the Ledger.

Of posting the Day-Book into the Ledger.

Some in posting, enter every article found in the Day-Book into the Ledger; but this method not only takes up too much room in the Ledger, but is too tedious in its operation. The better way is to collect together in one sum the several items on one page in the Day-Book, and enter the sum in the Ledger; observing to note in the Ledger, the page of the Day-Book, from whence the sum was taken; if the charges are but few in the Day-Book, they may be named in short in the Ledger; make some mark, in the margin of the Day-Book, for a post-mark,* that you may know when you have taken off each man's account.

* This (*) or any other mark may be used.

ALPHABET TO LEDGER A.

Make an alphabet, in which, enter each man's name, alphabetically, for the more readily finding each man's account in the ledger, naming in the alphabet the page or pages of the ledger, where the account may be found.

FORM OF THE ALPHABET.

		page.			page.
A.	Ayer Trustum,	1	G.	. . . .	
B.	Briant, David,	1	H.	. . . .	
C.	Calhound Enoch,	2	I.	Ingalls Thomas,	2
D.	Dwight Thomas,	2	J.	. . . .	
E.			K.	. . . .	
F.	Flint Abel, stock act.	1	L.	. . . .	

	Abel Flint, stock account, Dr.		
A.D. 1817. Dec. 1.	To the inventory of my estate, Dec. 1, 1817, . .	*Page* D.B.	$ cts. 2405 00
A.D. 1818. Dec. 1.	To the inventory of my estate, Dec. 1, 1818, . .		2780 25½

A. D. 1818.	Trustum Ayer, of Chester, Dr		
March.	To corn, potatoes, cheese and beans, . . .	2	$ cts. 13 12½
August.	" cider, corn, beans and team,	3	9 00½
December.	" vinegar, . . .	4	1 98
			$24 10½

	David Briant, of Exeter, Dr.		
A. D. 1818. March.	To flax, . . .	2	$3 20
August,	" horse and chaise to Portsmouth, . .	3	1 25
December.	" butter, cheese, and cider,	4	11 60
			16 05

Contra, Cr.

		page	Dls. cts.
A.D. 1817. December.	By sundry debts due from me Dec. 1, 1817, . .	D B. 1	852
	" neat of my estate, Dec. 1, 1817, after deducting all debts, ...		1553 00
			2405 00
	" sundry debts due from me Dec. 1, 1818, . .		773 80
	" neat of my estate, Dec. 1, 1818, after deducting all demands, . . .		2006 45
			2780 25½

Contra, Cr.

			Dls. cts.
A. D. 1818. March,	By labour, ...	2	6 00
	" a note for balance,		18 10½
			$24 10½
	Exeter, Jan. 1, 1819.—Then reckoned and settled all book accounts between the subscribers, and find due to Abel Flint, eighteen dollars and ten and a half cents, and settled the same by note. Trustum Ayer, Abel Flint.		

Contra, Cr.

A.D. 1818. December,	By note for balance of accounts, ...		$16 05
	Exeter, Dec. 1, 1818.—Then reckoned and settled all book accounts between the subscribers, and find due to Abel Flint, sixteen dollars — and five cents—paid the same by a note. David Briant, Abel Flint,		

Enoch Calhound, of Lee, Dr.

A.D. 1818.		page D B.	
March.	To flax, wood, corn, and potatoes, . . .	2	21 70
July.	" cider and pasturing horse,	3	11 00
			$32 70
	" balance due on settlement, Dec. 1, 1818, . .		$17 70

Thomas Dwight, of Chester, Dr.

A. D. 1818. March.	To eighteen and two thirds bushels salt, .	4	$14 00

Thomas Ingals, of Exeter, Dr.

A.D. 1818.			
July, Aug.	To corn, potatoes and pig,	3	3 75
Dec.	" corn, rye, potatoes, shoes, beans, and teaming,	4	34 62
			$38 37

Contra, Cr.

A. D. 1818. August.	By cash received by the hand of N. Briant, . .	*page.* D. B	Dls. cts. 15 00
	" balance due on settlement,	3	17 70
			$32 70
	Exeter, December 1,1818.—Then reckoned and settled all accounts between the subscribers, and find due to Abel Flint, 17 dollars seventy cents, to make even balance. Enoch Calhound, Abel Flint.		

Contra, Cr.

A. D. 1818. March.	By seven thousand shingles,	2	14 00

Contra, Cr.

A. D. 1818. Dec.	By balance on settlement,		38 37
	Exeter, December 20, 1818. Then reckoned and settled, and made even all accounts to this date. Thomas Ingals, Abel Flint.		$38 37

Inventory of goods, debts, and money belonging to me, Abel Flint, December 1, 1818, *and also of what I owe.*

	I have in cash on hand,	$125 00
100	Acres of land,	1000 00
6	Oxen,	180 00
30	Sheep,	60 00
2	Horses,	100 00
120	Bushels corn and grain,	120 00
	Pork, beef, and hay,	200 00
	Trustum Ayer's note,	18 10½
	David Briant's note,	16 05
	Balance of Enoch Calhound's account,	17 70
	Nathan Hoag's note,	650 00
	Interest on said note one year,	39 00
	Stephen Goodhue's note,	90 00
	Interest on said note one year,	5 40
	Stephen Beadle's note,	150 00
	Interest on said note one year,	9 00
		$2780 25½

Inventory of all the debts I owe, Dec. 1, 1818.

	Due to Peter Wilberforce,	650 00
	Interest on the same one year,	39 00
	Due to Seth Wright,	80 00
	Interest on the same one year,	4 80
		773 80

Neat of my estate, $2005 80.

FORM OF AN ACCOUNT CURRENT.

Dr. Mr. Samuel Gorham of Portsmouth, in account current with John Laird of Exeter, *Cr.*

A. D. 1832.			A. D. 1832.		
June 20.	To 6 bbls. flour, at $10 pr. bl.	$60 00	Sept. 19.	By cash,	$61 20
July 11.	" 1 cwt. sugar at $15 per cwt.	15 00	Nov. 11.	By an order on Mr. A. B.	15 16
do. 19.	" cash paid J. Dirgan per order	10 00	Dec. 21.	By cash by the hand of J. Orr,	29 76
Aug. 20.	" sundries as per bill,	26 93	do. 24.	By repairing my chaise,	21 70
do. 29.	" goods deliv. to A.B. pr. order	29 25			
Sept. 6.	" 1 bl. rum, 32½ gal. at 1 20 cts.	39 00	A. D. 1833.		
Oct. 10.	" 42 gal. molasses at 56 cts.	23 52	Jan. 15.	By cash by the hand of J. Orr,	101 63
Dec. 21.	" 2½ cwt. of iron, at 5 50 cts.	13 75			$229 45
do. 29.	" 60 lbs. sugar, at 10 cts.	6 00			
		$229 45			

When a person is furnished with his account current, the various charges should be specified, and the dates when the articles were delivered, when the articles are numerous, some accountants make but one charge in the account current, refering to an annexed account of the several articles included.

Exeter, 15th January, A. D. 1833.

Errors excepted.

JOHN LAIRD.

TABLES of the value of the *Gold Coins* of GREAT BRITAIN, FRANCE and SPAIN, according to the Act of Congress of April 29, 1816.

A Table shewing the weight of any sum of English or Portuguese Gold, from one to ninety dollars.

Dls.	dwts	Grs.	Dls.	dwts	Grs.	Dls.	dwts	Grs.
1	1	3	7	7	21	40	45	00
2	2	6	8	9	00	50	56	6
3	3	9	9	10	3	60	67	12
4	4	12	10	11	6	70	78	18
5	5	15	20	22	12	80	90	00
6	6	18	30	33	18	90	101	6

A Table shewing the weight of any required sum of French Gold, from one to ninety dollars.

Dls.	dwts	Grs.	Dls.	dwts	Grs.	Dls.	dwts	Grs.
1	1	4	7	8	1	40	45	20
2	2	7	8	9	4	50	57	7
3	3	11	9	10	8	60	68	18
4	4	14	10	11	11	70	80	5
5	5	18	20	22	22	80	91	16
6	6	21	30	34	9	90	103	3

A Table shewing the weight of any sum of Spanish Gold, from one to ninety dollars.

Dls.	dwts	Grs.	Dls.	dwts	Grs.	Dls.	dwts	Grs.
1	1	5	7	8	8	40	47	15
2	2	9	8	9	13	50	59	13
3	3	14	9	10	17	60	71	10
4	4	18	10	11	22	70	83	8
5	5	23	20	23	19	80	95	6
6	6	04	30	35	17	90	107	3

USEFUL FORMS OF WRITINGS.

Form of a Bill of Exchange.

Portsmouth, N. H., April 1, 1832.

Exchange for £650 sterling.

At sixty days sight of this my first of exchange (second and third of the same tenor and date not paid) pay to W. Richards, or order, six hundred and fifty pounds sterling with, or without advice from

Your humble servant,
JAMES JACKSON.

Messrs. Corridon & Cutler,
Merchants, Bristol.

OF NOTES.

Form of a Note by Person.

Newburyport, April 1, 1832.

For value received, I promise to pay to John Laird, or order,* three hundred and sixty-seven dollars, and twenty-five cents, on demand, with interest. Witness my hand,
SETH BROOKINGS.

$367 25.

* A note is not negotiable unless the words, *or order*, or the words, *or bearer*, are inserted. If it is written to pay J. L. or order, then J. L. may write his name on the back side, and sell it to A. (or any one else) then A. who buys the note may call on Seth Brookings for the payment of the same, and if he is unable, or neglects to pay it, A. may recover the same of J. L. If the note were written to pay John Laird, or bearer, then any person may recover the same of Seth Brookings.

Further observations concerning notes.

All notes are either payable on demand, or at a certain time mentioned in the note; all notes payable at a certain time are on interest as soon as they become due, although interest is not mentioned in the note; a note payable on demand, will draw interest after a demand of payment is made, if interest is not mentioned in the note. If the note is given for any commodity,—as corn, lumber, &c., payable at the expiration of a certain time, and the signer of the note does not hold the article ready, or does not pay it at the time it is demanded, the holder of the note is not obliged to take the article, but may demand and recover the value of the article in money.

Another form of a Note.

I promise to pay Aaron Kimball, or order, one hundred dollars, on demand with interest, after ninety days, for value received at Exeter this first day of April, A. D. 1832.
Witness my hand,

ROGER PERKINS

$100 00

Form of a Note given by two or more persons jointly and severally.

Exeter, April 1, 1832.

For value received, we the subscribers promise jointly and severally to pay Andrew Fairbanks, or order, one hundred dollars in one year from the date, with interest.
Witness our hands.

JAMES RUSH,
THOMAS SUTTON.

$100 00

OF RECEIPTS.

The form of a Receipt for money received on account.

Concord, April 10, 1832.

Received of Isaac Sewall, sixty-eight dollars, on account.

LEMUEL LOVEJOY.

The form of a Receipt for an endorsement on a Note.

Augusta, April 10, 1822.

Received from David Goodhue, one hundred and eighty dollars, by the hand of James Osgood, which is endorsed on his note of June 12, 1832.

W. SPEARE.

The form of a Receipt in full of all accounts.

Portland, April 10, 1832.

Received of Stephen Hammons, six dollars, in full of all accounts.

JOHN BIRDSEYE.

*The form of a Receipt in full of all demands.**

Kennebunk, April 10, 1832.

Received of Lemuel Osgood, six dollars, in full of al demands to this date. ENOCH GOODROY.

OF ORDERS.

Waterville, March 10, 1833.

For value received,† pay to Stephen Hingham, ten dol lars, and place the same to my account.

To Mr. Alfred Goodpay. ABNER DREW.

ANOTHER FORM.

Waterville, August 10, 1833.

For value received, pay Nathan Livingston, six dollars and this with his receipt shall be your discharge.

To Mr. Stephen Skinner. LEMUEL NEEDY.

OF DEEDS.

Form of a Warrantee Deed.

Know all men by these presents,—That I, William Tru ty of Exeter, county of Rockingham, state of New-Hamp shire, gentleman, in the consideration of the sum of si hundred dollars, paid me by William Hadley of the sam Exeter, yeoman, the receipt whereof I do hereby acknow ledge, do hereby give, grant, bargain, sell, and convey unt the said William Hadley, his heirs and assigns foreve (here insert the premises, particularly describing the town county and state where the land lies, and describing th boundaries, on whom joining, &c.) To have and to hol the said granted and bargained premises with the privil

* There is a distinction to be made between a receipt given in full of a accounts, and one given in full of all demands, the former will cut of a counts only, but the latter will cut off all obligations, and prohibit the rig of an action.

† The words, *For value received*, should always be mentioned in order, then if A should give B an order on C for a certain sum of mone and C should not accept the order, B could sue A for the order,-and recu er the same, as well as if it had been a note of hand.

ges, and appurtenances thereof, to him the said William Hadley, his heirs and assigns forever. And I, the said William Trusty for myself, my heirs, executors, and administrators, do covenant with the said William Hadley, his heirs and assigns, that I am lawfully seized in fee of the premises, that they are free of all incumbrance; that I have good right to sell and convey the same to the said William Hadley to hold as aforesaid.

And that I will warrant and defend the same to the said William Hadley, his heirs and assigns forever, against the lawful claims and demands of all persons whomsoever.

In witness whereof I have hereunto set my hand and seal, this tenth day of July, A. D. 1833.

Signed, sealed and delivered
In the presence of us, WM. TRUSTY. [L. S.]
Nathan Lovejoy,
Samuel Langley.

MORTGAGE DEEDS.

[A Mortgage Deed is written the same as a warrantee deed, until you come to the word *whomsoever;* then the proviso, or condition of the mortgage should follow, in the following form, viz.]

Provided nevertheless, That if I, the said William Trusty, my heirs, executors, or administrators shall well and truly pay to the said William Hadley, his heirs, and executors, administrators or assigns, the full and just sum of—— dollars on or before the——day of——next, (or which will be in the year of our Lord ——) with interest for the same until paid: then this deed (as also a certain bond, or note, as the case may be, bearing even date with these presents, given by me to the said William Hadley, conditioned to pay the same sum and interest at the time aforesaid) shall be void; otherwise shall remain in full force and virtue:

In witness whereof I have, &c.

The form of a Quitclaim Deed.

Know all men by these presents, That I, Joseph Doe of Durham, county of Strafford, and State of New-Hamp-

shire, yeoman, in consideration of four hundred dollars to me paid by Francis Cady of Warner, county of Hillsborough, State of New-Hampshire, gentleman, the receipt whereof I do hereby acknowledge, have remissed, released, and forever quitclaimed, and do by these presents, remiss, release and forever quitclaim unto the said Francis Cady, his heirs and assigns forever, (here the premises must be described; and the description should always point out where the land lays, on whom it is joining, the length of the lines and the point of the compass they run should be mentioned) to have and to hold the same, together with all the privileges and appurtenances thereunto belonging, to him the said Francis Cady, his heirs and assigns forever.

In witness whereof I have hereunto set my hand and seal this tenth day of August, A. D. 1833.

Signed, sealed and delivered
in presence of us, JOSEPH DOE. [L. S.]
William Stebbins,
Nicholas Williams.

Form of an agreement.

Articles of agreement made and concluded the tenth day of April, one thousand eight hundred and thirty-three, by and between Thomas Smith of Dover, county of Strafford, State of New-Hampshire, on one part, and Aaron Watkins of Warren, county of Grafton, State of New-Hampshire, on the other part, *witnesseth:*

That the said Thomas Smith for the consideration of one hundred dollars worth of harness leather, to be paid by Aaron Watkins as hereafter mentioned, hath agreed, and doth hereby covenant and agree to make, or cause to be made, a good and warranted chaise with standing top, every part of which shall be made and finished in a good and workmanlike manner, together with a good plated harness well made, and every way suitable for a good chaise; which chaise and harness, the said Thomas Smith doth hereby agree to have made and ready to be delivered to said Watkins, on the sixteenth day of August next.

And the said Aaron Watkins on his part, doth hereby covenant and agree to pay, or cause to be paid, one hundred dollars worth of harness leather, suitably tanned and

curried, on the sixteenth day of August next, delivered at Dover.

To the true and faithful performance of the several covenants and agreements aforesaid, the parties aforesaid do hereby respectively bind themselves and their respective heirs, executors, and administrators each to the other, his executors and administrators in the penal sum of ——, in testimony whereof they have hereunto interchangeably* set their hands and seals—the day and year above written.

THOMAS SMITH. [L. S.]

Signed, sealed and delivered
in presence of us,
David Hadley, AARON WATKINS. [L. S.]
John Hastings.

Form of a will with the devise of a real and leasehold estate.

In the name of God, Amen.—I, A. B. of ——— being weak in body, but of sound and perfect mind and memory, (or you may say thus, considering the uncertainty of this mortal life, and being of sound and perfect mind and memory) blessed be Almighty God for the same, do make and publish this my last will and testament, in manner and form following, viz.—

First, I give and bequeath unto my beloved wife T. B. the sum of ———dollars; I do also give and bequeath unto my eldest son, H. B. the sum of—— dollars; I do also give and bequeath unto my two younger sons, J. B. and F. B., the sum of—— dollars a piece; I also give and bequeath unto my daughter-in-law, S. H. single woman, the sum of ——dollars; which said several legacies or sums of money, I will and order shall be paid the said respective legatees, within six months after my decease. I further give and bequeath to my said eldest son H. B., his heirs and assigns, *all* that my messuage or tenement, situate, lying and being in ——, together with all my other freehold estate whatsoever; to hold to him the said H. B. his heirs and assigns

* If the word *interchangeably* is used, each party must have a draught of the agreement, which is the proper method, and the draught that Thomas Smith has, must be signed by Aaron Watkins first, and the one which Aaron Watkins has, Thomas Smith must sign first.

ferever. And I hereby give and bequeath to my said younger sons, J. B. and F. B., all my leasehold estate of and in all those messuages or tenements, with the appurtenances, situate in ——, equally to be divided between them. And lastly, as to all the rest, residue and remainder of my personal estate, goods and chattels, of what kind and nature soever, I give and bequeath the same to my said beloved wife, T. B. whom I hereby appoint sole executrix of this my last will and testament; hereby revoking all former wills by me made.

In witness whereof, I have hereunto set my hand and seal the —— day of —— in the year of our Lord ——.

Signed, sealed, published and declared by the above named A. B. to be his last will and testament, in the presence of us, who have hereunto subscribed our names as witnesses, in the presence of the testator. A. B. [L. S.]

SAMUEL TRUSTY,
WILLIAM SUTTON.

Form of an Indenture to bind an apprentice, to learn a trade till twenty-one.

This indenture witnesseth, That J. B. of Dover, in the county of Strafford, State of New-Hampshire, hath put and placed; and by these presents doth put and bind out his son D. B., and the said D. B. doth hereby put, place and bind out himself, as an apprentice to A. D. to learn the art, trade or mystery of printing, the said D. B. after the manner of an apprentice, to dwell with and serve the said A. D. seven years, which will be in the year of our Lord one thousand eight hundred and forty one, at which time the said apprentice, if he shall be living, will be twenty one years of age; during all which time or term, the said apprentice his said master well and faithfully shall serve; his secrets keep; his lawful commands every where at all times readily obey; he shall do no damage to his said master, nor wilfully suffer any to be done by others; and if any to his knowledge be intended, he shall give his master seasonable notice thereof. He shall not waste the goods of his said master, nor lend them unlawfully to any; at cards, dice or any other unlawful game he shall not play; forni-

cation he shall not commit, nor matrimony contract during the said term; taverns, ale houses, or places of gaming he shall not haunt or frequent; from the service of his said master he shall not absent himself; but in all things and at all times he shall carry and behave himself as a good and faithful apprentice ought, during the whole time or term aforesaid.

And the said A. D. on his part, doth hereby promise, covenant and agree to teach and instruct the said apprentice, or cause him to be taught and instructed in the art, trade or calling of a printer, by the best way or means he can, and also to teach and instruct the said apprentice, or cause him to be taught and instructed, to read and write, and cypher as far as the Rule of Three, if the said apprentice be capable to learn; and shall well and faithfully find and provide for the said apprentice, good and sufficient meat, drink, clothing, lodging, and other necessaries fit and convenient for such an apprentice, during the term aforesaid, and at the expiration thereof, shall give unto the said apprentice, two suits of wearing apparel, one suitable for the Lord's day, and the other for working days.

In testimony whereof, the parties have hereunto interchangeably set their hands and seals this first day of April, in the year of our Lord one thousand eight hundred and thirty-three.

Signed, sealed and delivered in presence of us witnesses,
WILLIAM ROLLY,
ICHABOD MACE.

A. D. (seal.)
J. B. (seal.)
D. B. (seal.)

An indenture for binding out a poor child, by the overseers of the poor.

This indenture witnesseth, that B. F. W. P. and S. W. overseers of the poor of the town of ——, in the county of ———, by the virtue of a law of the State of ———, in such cases made and provided, have placed and by these presents do place and bind out as an apprentice, a poor child named ———, son of ———, of said town of ———, who is lawfully settled in and become chargeable to said town, and who is thought by said overseers to be unable to maintain him, unto—— to learn, &c. (here proceed as in the first form until you come to the words "working days") and such other instructions, benefit and allowance, either

within or at the end of the term, as the overseers may see fit and reasonable.

In testimony whereof, the said parties have hereunto interchangeably set their hands and seals the —— day of—— A. D. ——.

Signed, sealed and delivered in presence of us witnesses,
Walter King,
Oliver Ray.

B. F. (seal.)
W. P. (seal.)
S. W. (seal.)

A short form of a lease from one to another.

This indenture made the eighth day of April, A. D. one thousand eight hundred and thirty two, by and between S. B. of Poplin, county of Rockingham, State of New-Hampshire, on one part, and W. P. of the same Poplin, on the other part, *witnesseth*, that the said S. B. for the consideration hereafter mentioned, hath demised, granted, and farm letten, and doth hereby demise, grant and to farm let unto the said W. P. his heirs and assigns a certain three story dwelling house, situated in the same Poplin. known by the name of the Poplin Hotel, with all the privileges and appurtenances thereunto belonging; to have and to hold the said demised premises with their appurtenances for and during the term of six years from the date hereof, *then* to be complete and ended.

And the said W. P. for himself, his heirs, executors, and administrators, doth covenant and agree to pay or cause to be paid in quarterly payments, two hundred dollars yearly, and at the expiration of the six years, peaceably to yield and release all claim in and to said premises.

In witness whereof, the parties have hereunto interchangeably set their hands and seals, the day and year above written.

Signed, sealed and delivered in presence of us,
Ichabod Chase,
John Lingan.

S. B. (seal.)
W. P. (seal.)

OF RETURNS ON WRITS, &c.

A return to be made upon a writ of attachment.

Brentwood, April 12, 1833.

Pursuant to the within writ I have attached a—— the property of the within named A. B. value ——— and left a summons at his dwelling house, or place of his last and usual abode (or if you give the summons to him, say, and delivered him a summons) according to law.

H. C. K. Constable.

Fees,	9 miles travel,	.54
"	service,	.23
		$0.77

Return to be made upon a special writ.

Brentwood, April 13, 1833.

Pursuant to the within precept I have taken the body of the within named A. B. and taken C. D. bail for his appearance at court, as within mentioned (if the prisoner is committed to jail the return should read thus,) and have committed him to gaol in —— that he may be holden for his appearance as within mentioned.

H. C. K. Constable.

Fees,	10 miles travel,	.60
"	service.	.23
		$0 83

A return to be made upon a warrant.

Brentwood, April 12, 1833.

Pursuant to the within warrant, I have apprehended the body of the within named B. W. and have him before S. T. esquire, one of the justices of the peace for said county, for the purpose within mentioned. I have also summoned the within named C. D. and E. F. to appear as witnesses.

S. D. Constable.

Fees,

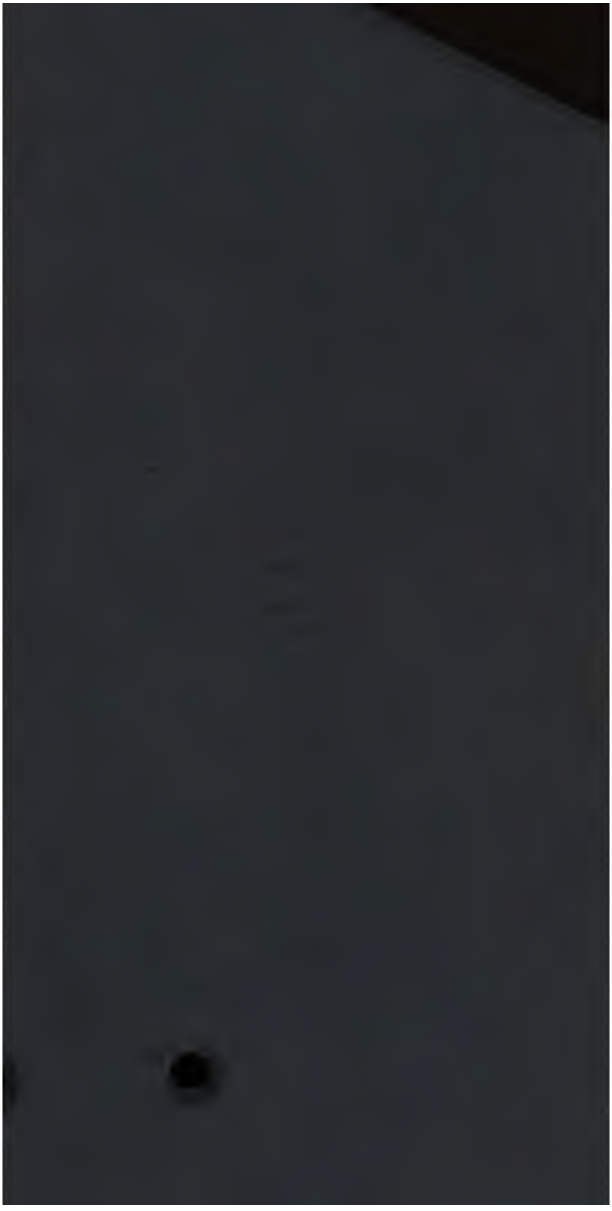
stereotyped edition. The following are selected:

Extract from the New-Hampshire Courier, Nov. 1853.

"We have carefully examined Welch's Improved Arithmetic, published by Lincoln, Edmands & Co., Boston, and most cheerfully recommend it to the public as a work admirably well calculated for the use of the public schools in this State.

"Having carried some eighty or ninety youth through Welch's Arithmetic, we claim to know something of the work; and we have seen and felt the need of the improvements, which have been made to it by the Revisor, to whose labors the public will be in part indebted for one of the best Arithmetics now to be obtained.

"We are pleased to observe that the mechanical part of the book has not been neglected; it is printed on fair type and good paper, two things very often neglected in school books. We hope teachers will examine the above work for themselves, before they recommend to their scholars the purchase of other Arithmetics."

From the Portsmouth Journal, Nov. 16, 1853.

"We have examined, with some attention, the claims of this elementary treatise on one of the most important branches of early education, and are prepared to say that we know of no work of the kind which appears better adapted to the minds of young pupils, in acquiring a good knowledge of arithmetic, than the one before us. Every rule is put down in its appropriate place, and